Recent Advances in Agglomerated Multigrid

Hiroaki Nishikawa, Boris Diskin,

National Institute of Aerospace

James L. Thomas, Dana P. Hammond

NASA Langley Research Center

NATIONAL INSTITUTE OF AEROSPACE

AIAA Aerospace Sciences Meeting

Wednesday January 9, 2013

Joint Fluid Dynamics and Meshing/Visualizaton Session

Grapevine, TX

Purpose

Improve efficiency of Reynolds-Averaged Navier-Stokes (RANS) simulations for complex-geometry and complex-flow applications

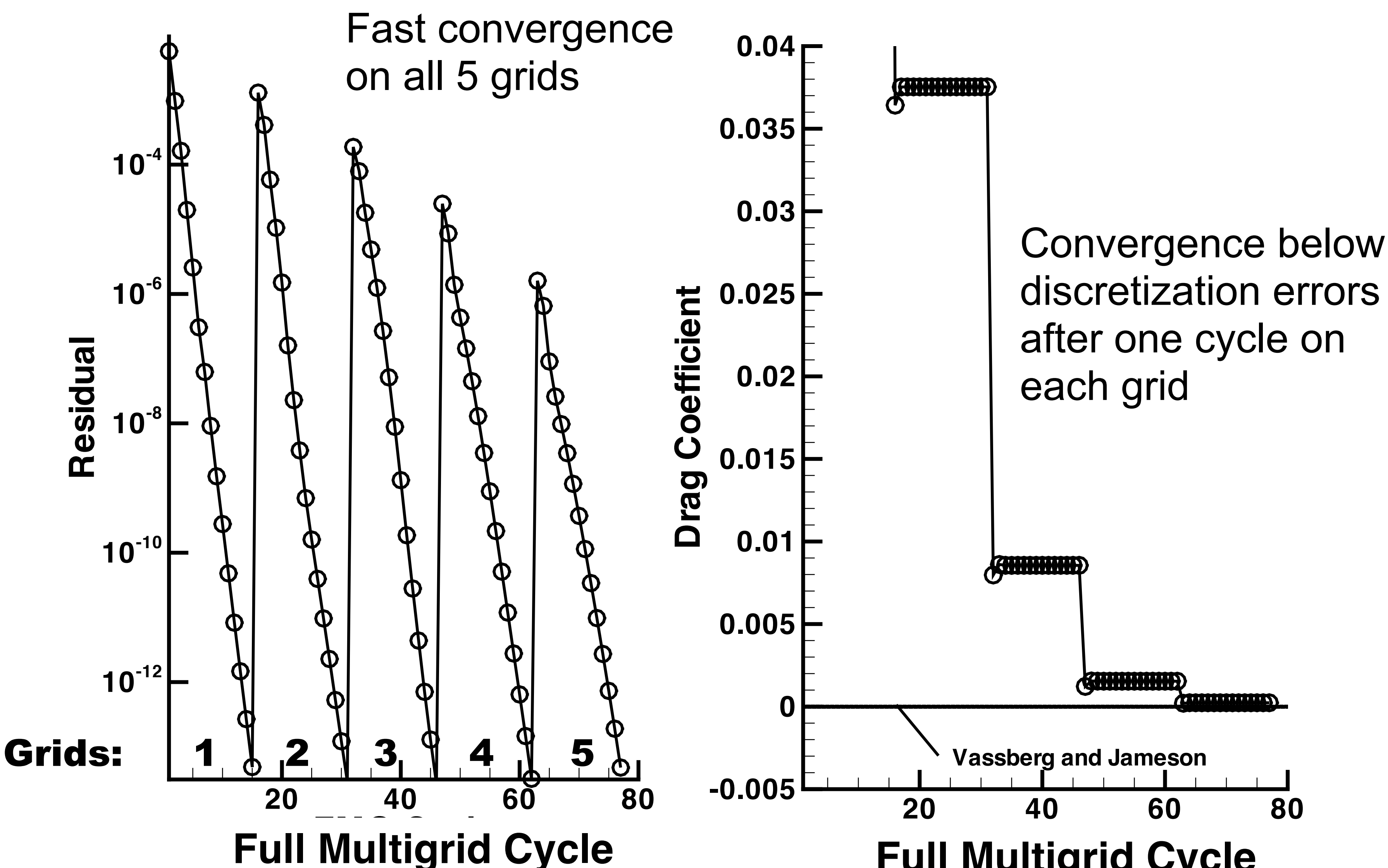
- Unstructured compressible flow method (FUN3D)
- Spalart-Allmaras 1-equation turbulence model
- Multigrid with agglomerated coarse grids

Previous Work

- Agglomerated multigrid methods well suited to unstructured grids
  - Nonlinear multigrid from 1977
  - Agglomeration methods from 1987
  - Speedups observed but gains fall short of gains for inviscid or laminar flows
- Current approach extends hierarchical multigrid method (1999) to unstructured grids
  - Assessed defect correction for compressible Euler (2010)
  - Developed agglomeration method preserving features of geometry (2010)
  - Critically assessed multigrid for diffusion (2010), identified by Venkatakrishnan (1996) as weakest part of agglomeration
  - Applied to complex inviscid/laminar/turbulent flows (2010/2011)

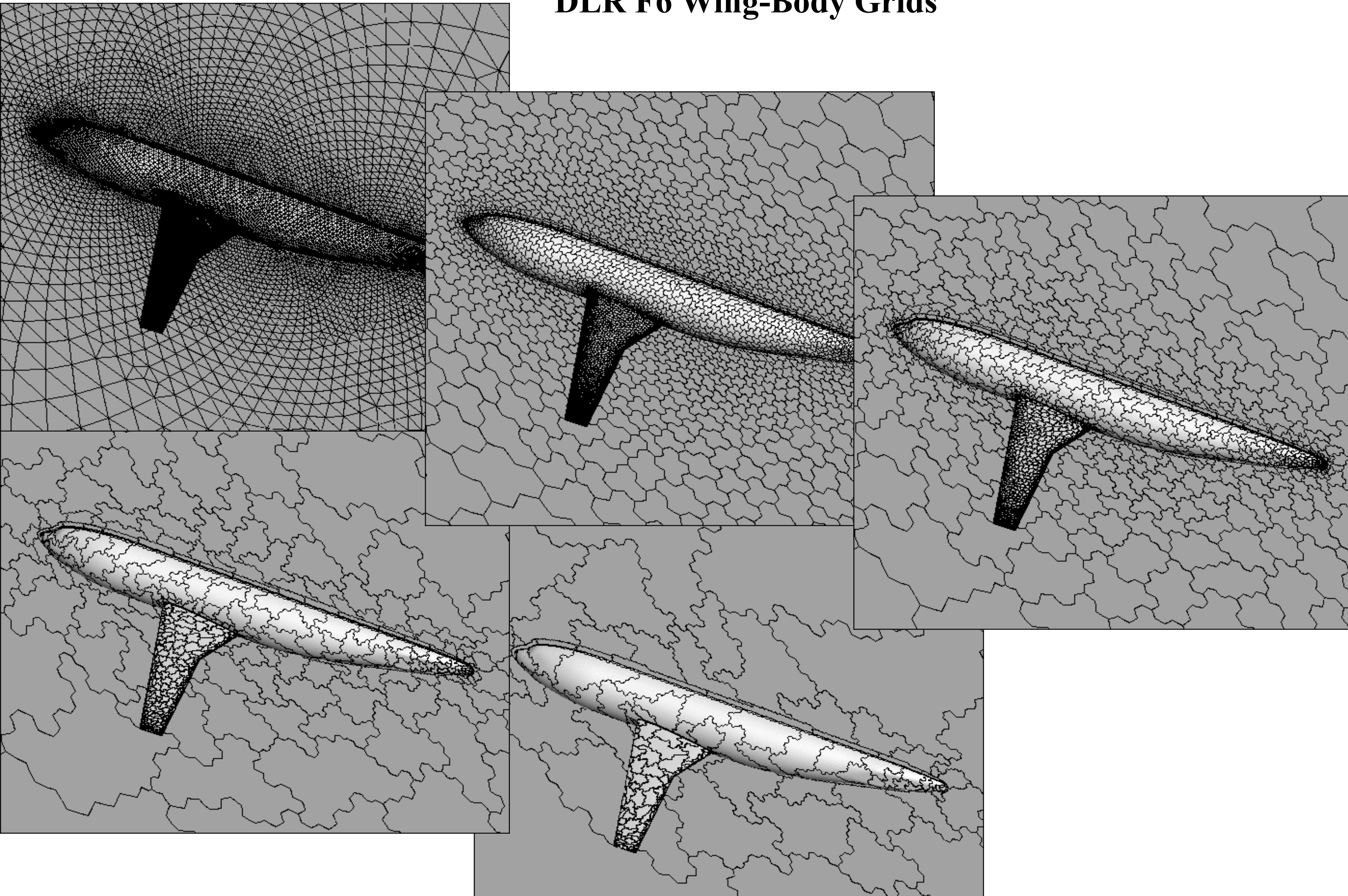
Multigrid for Euler (2011)

NACA 0012 airfoil; M= 0.5; Alpha =1.25



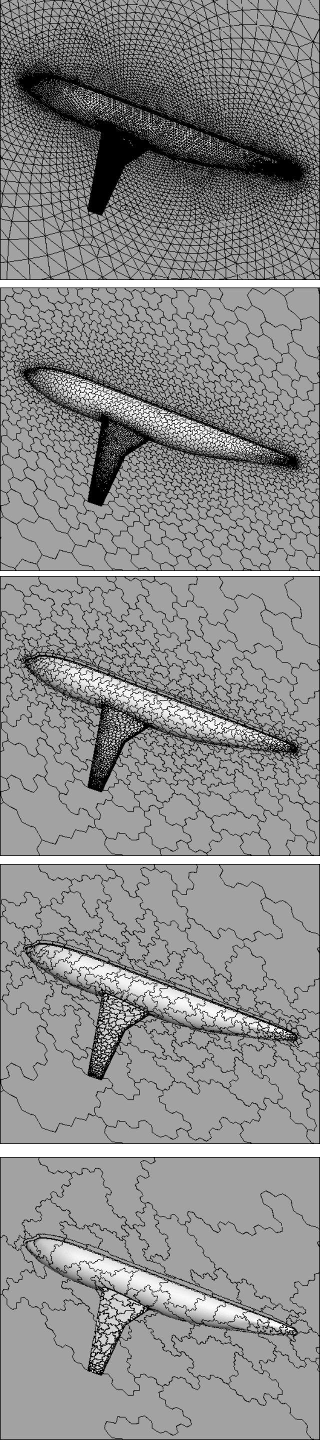
Multigrid for Diffusion (2010)

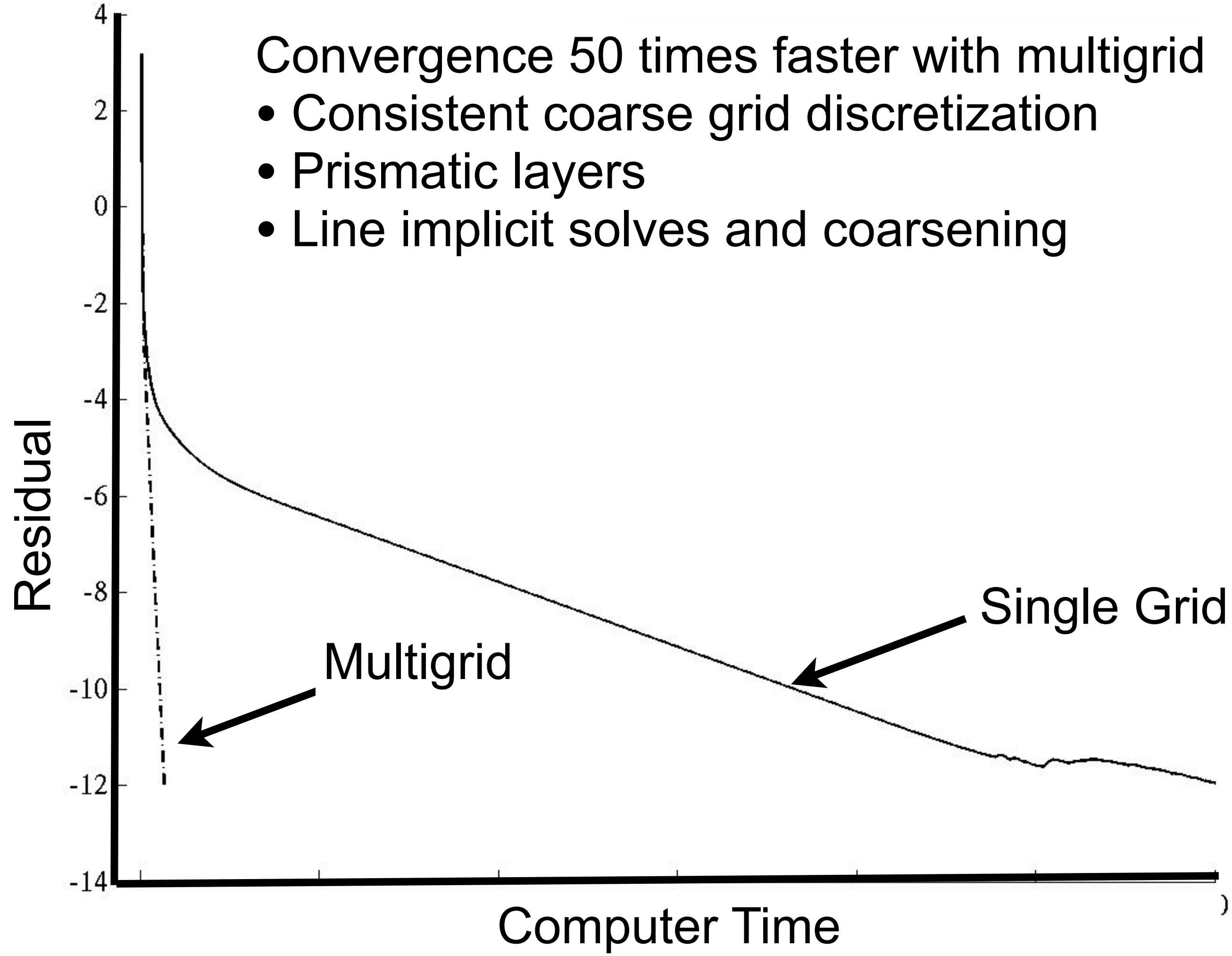
DLR F6 Wing-Body Grids



Multigrid for Diffusion (2010)

DLR F6 Wing-Body

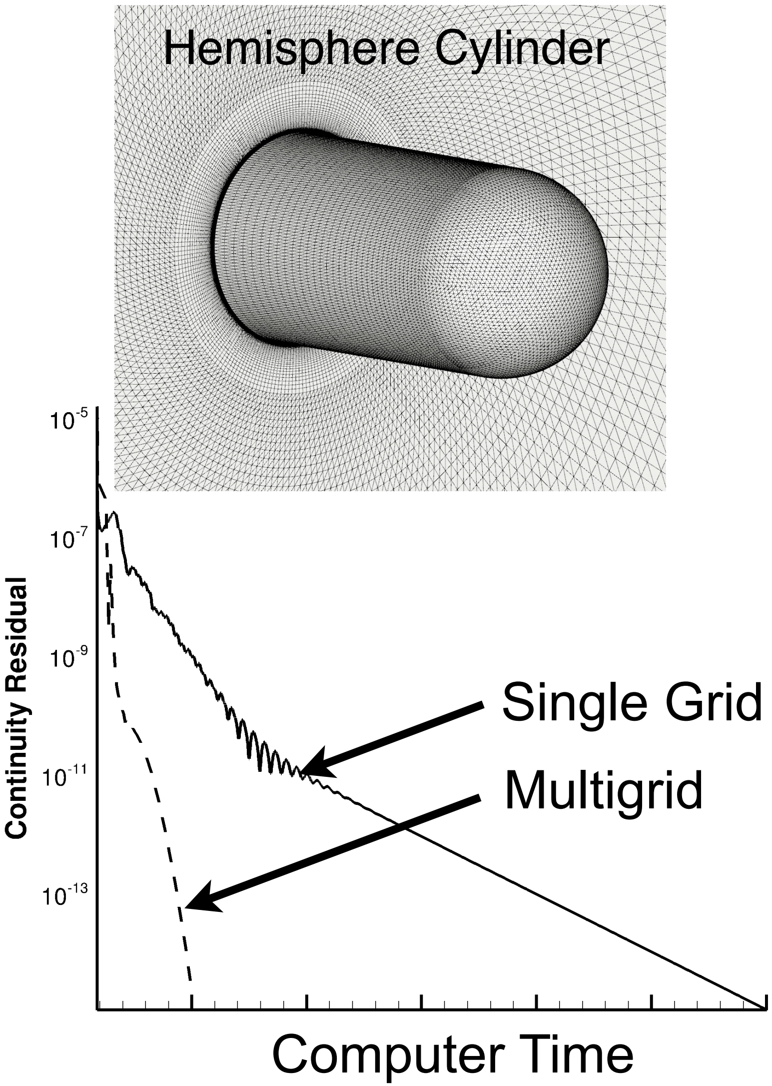




3D Agglomerated Multigrid (2011)

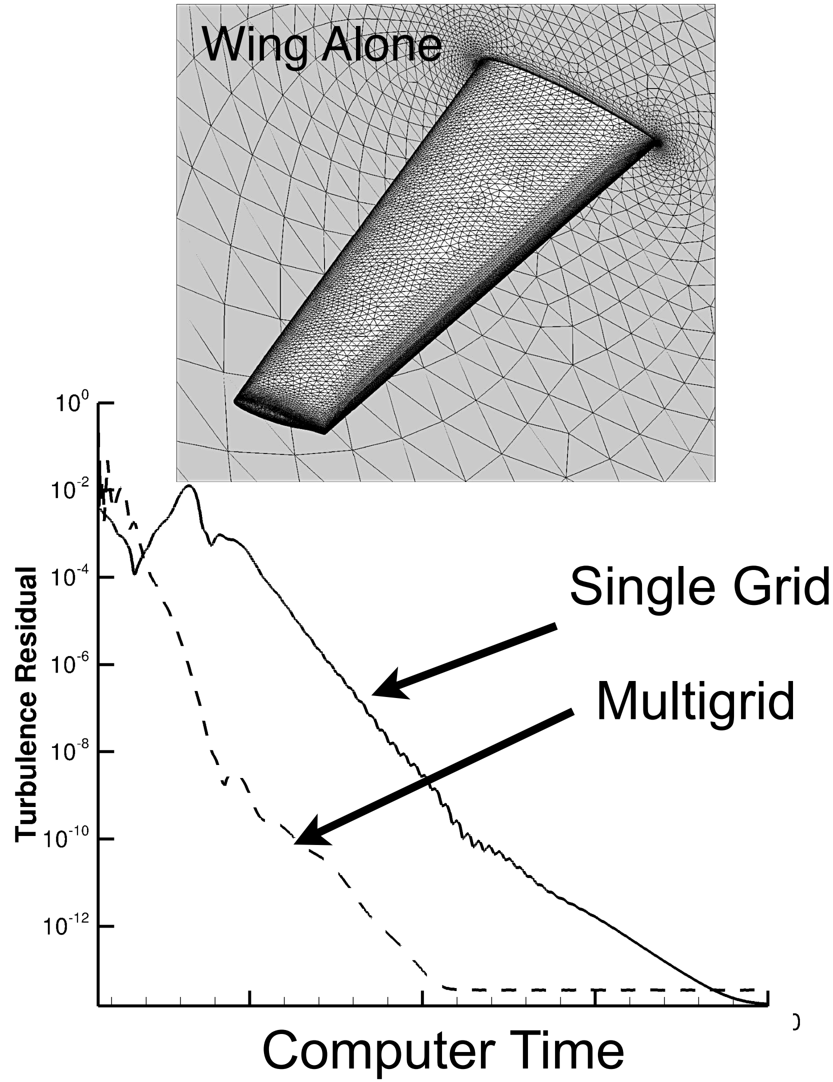
Subsonic Laminar

20 Million nodes



Transonic RANS

12 Million nodes



Previous Work

- Agglomerated multigrid methods well suited to unstructured grids
  - Nonlinear multigrid from 1977
  - Agglomeration methods from 1987
  - Speedups observed but gains fall short of gains for inviscid or laminar flows
- Current approach extends hierarchical multigrid method (1999) to unstructured grids
  - Assessed defect correction for compressible Euler (2010)
  - Developed agglomeration method preserving features of geometry (2010)
  - Critically assessed multigrid for diffusion (2010), identified by Venkatakrishnan (1996) as weakest part of agglomeration
  - Applied to complex inviscid/laminar/turbulent flows (2010/2011)
- Overall status
  - Uniformly successful for laminar and inviscid simulations
  - Limited success for RANS simulations

Purpose - Revisited

Improve efficiency of Reynolds-Averaged Navier-Stokes (RANS) simulations for complex-geometry and complex-flow applications

- Unstructured compressible flow method (FUN3D)
- Spalart-Allmaras 1-equation turbulence model
- ~~Multigrid with agglomerated coarse grids~~

Single grid   structured (regular coarsening)

"When there is a difficult question that you cannot answer, there is also a simpler question that you cannot answer" (Achi Brandt)

Solving turbulence equations to zero residuals is one such question

Why Such a Difficult Question?

Spalart-Allmaras Model
with negative turbulence variable *provisions* (2012)

- Nonlinear diffusion and source terms
- Production source terms associated with exponentially growing solutions are eventually balanced by other terms
  - Reduce diagonal positivity
  - Require gradients of mean flow variables
- RANS necessitates high aspect ratio, highly stretched grids
  - 3D grid refinements stretch computer resources
  - Questionable accuracy on coarse grids
- Discrete solutions depart from positivity of differential equations (*provisions* date to 1999)
  - Negative turbulence variable => zero eddy viscosity
  - Steady 3D flows with negative turbulence variable in far wake regions
  - Some first-order 3D cases are harder to solve than second-order

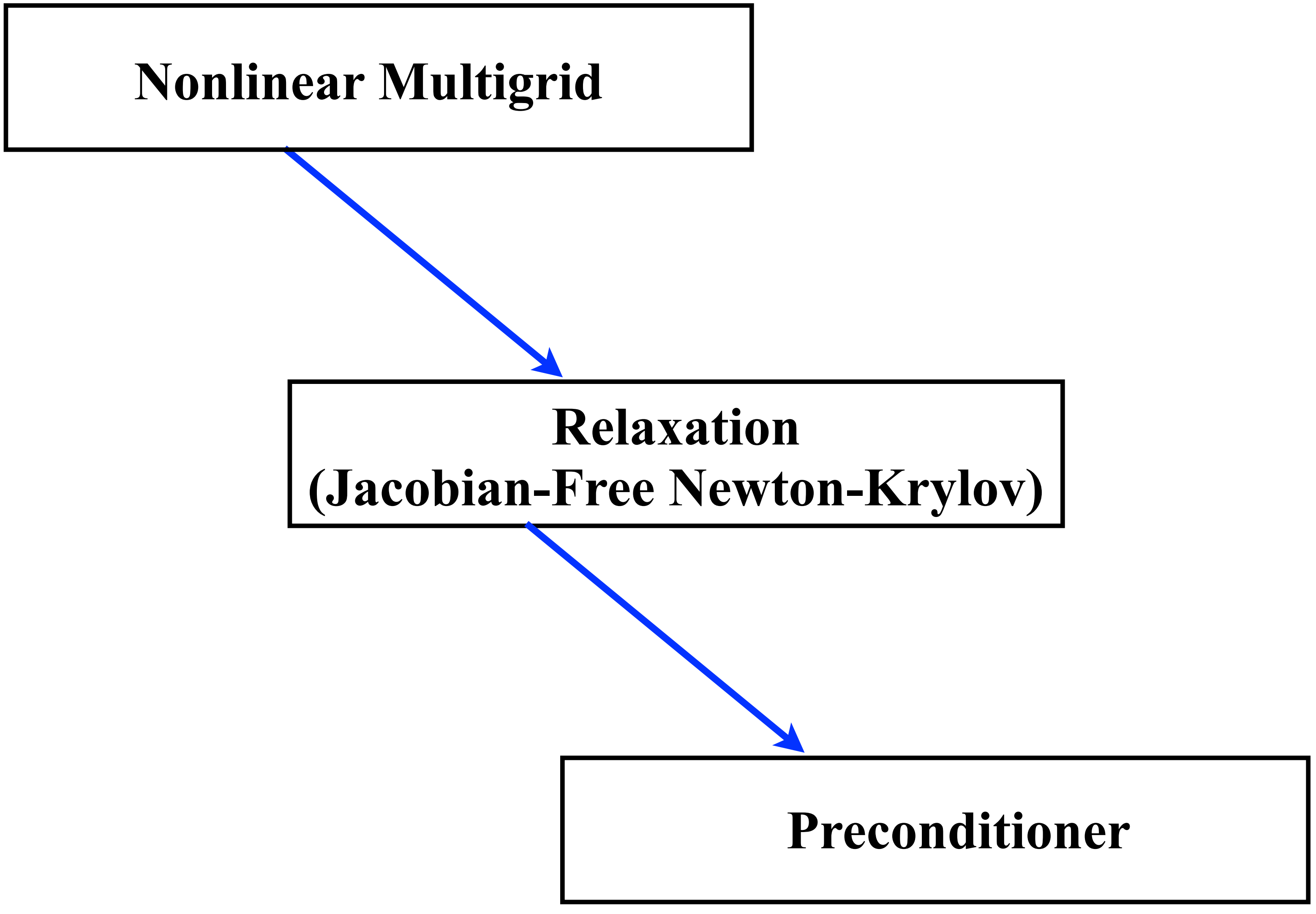
Remainder of Talk

- Hierarchical solver with adaptive time step
- Agglomerated and structured multigrid comparisons
- Transonic 3D wing-body computations
- Concluding remarks

Contributions of this Paper

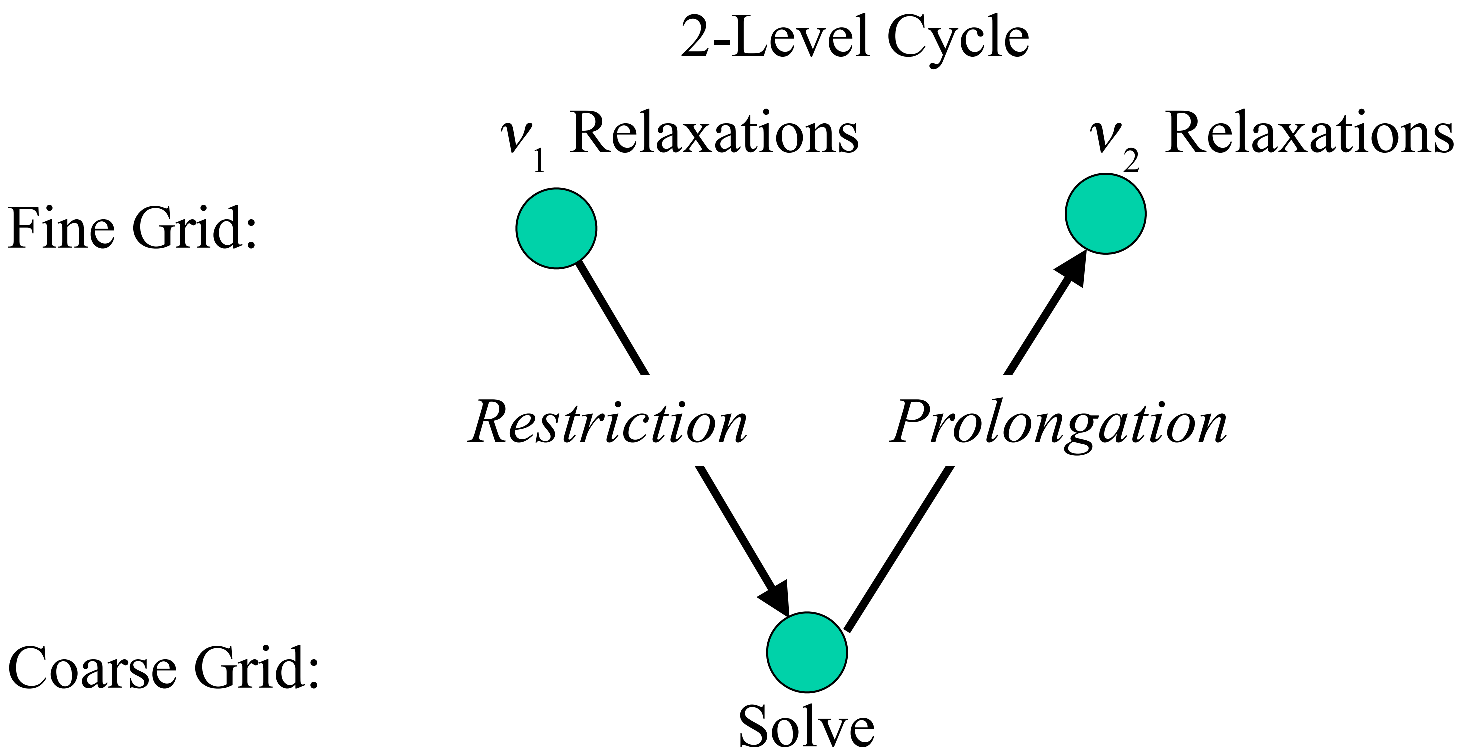
- Improved hierarchical solver with adaptive pseudo-time step
- Assessment of current technology with systematic tests
  - Increasing complexity and grid refinement
  - Structured and agglomerated grids
- Parallelization improvements
- Eliminated degenerate least-square stencils (fine/coarse grids)
- Term-by-term formation of Jacobians
- Improved discretization in line-implicit regions

Hierarchical Solver



Nonlinear Multigrid

Full Approximation Scheme V(ν_1,ν_2) Cycle



- Recursive application of 2-grid cycle
- Mean flow and turbulence relaxed at every level
  - Loosely-coupled (meanflow, then turbulence)
  - Tightly-coupled

Jacobian-Free Newton Krylov

Generalized Conjugate Residual (GCR)

Target update equation is full linearization with adaptive time step (CFL)

$$\left(\frac{V}{\Delta\tau}+\frac{\partial R}{\partial Q}\right)\delta Q=-R \quad ; \quad Q=Q+\delta Q$$

$V \equiv$ volume R $\equiv$ residual

$\Delta\tau \equiv$ time step Q $\equiv$ solution

Full linearization is through Jacobian-free matrix evaluation

- Real-valued (uncertainties from round-off and constrained growth)
- Complex-valued (nominally exact)

GCR combines preconditioner directions to minimize residual, generally with loose tolerance and a few projections

$$\left\|\left(\frac{V}{\Delta\tau}+\frac{\partial R}{\partial Q}\right)\delta Q+R\right\| \le f\left\|R\right\| \qquad 0.5 \le f \le 0.98$$

GCR often stabilizes divergent preconditioner subiterations

Preconditioner

Jacobian Approximations with Subiterations

Target update (direction) equation is approximate linearization with adaptive time step

$$\left(\frac{V}{\Delta\tau}+\widehat{\frac{\partial \mathrm{R}}{\partial \mathrm{Q}}}\right)\widehat{\delta Q} = -R$$

where linearization is approximate, e.g., on primal grids:

- first-order accurate inviscid terms (defect correction)
- exact viscous terms

Alternating multicolor point-implicit and line-implicit subiterations, solving with loose tolerance

$$\left\|\left(\frac{V}{\Delta\tau}+\widehat{\frac{\partial R}{\partial Q}}\right)\widehat{\delta Q} + R\right\| \le f\left\|R\right\| \qquad 0.5 \le f \le 0.98$$

Can often take many subiterations to meet even minimal tolerance of $f \le 0.98$

Pseudo-Time Step (CFL) Adaptation

- Motivated by recent work of Allmaras et al. (2011) on robust Newton solver
  - Direct linear solver
  - CFL low in highly nonlinear regions and where allowable changes exceeded
  - CFL eventually high with quadratic convergence
- Current approach evolving
  - CFL reduced whenever linear systems are having difficulty reaching loose tolerance with minimal GCR projections
  - CFL reduced and null update nulled whenever linear systems converging but update extremely large
  - Otherwise, similar to Allmaras et al. but without quadratic convergence

Evolution of Target CFL Strategy

Strategy	CFL Target	Preconditioner
Clydesdale	200	Dissipative Defect Correction
Quarter Horse	200	Consistent Defect Correction
Thoroughbred	10,000	Consistent Defect Correction

- Enabled by CFL adaptation

- Enabled by GCR

Turbulent Test Cases (SpeedUp over Single Grid)

V(3,3) ; CFL=200 (Quarter Horse CFL Target)
Based on Convergence to Machine Zero Residuals

	Geometry	Finest Grid Nodes	Agglomerated Multigrid SpeedUp
2D	Bump in a Channel	4K	3x
2D	RAE Airfoil	98K	3x
2D	Flat Plate	209K	9x
2D	NACA 0012 Airfoil	919K	8x
2D	Hemisphere Cylinder	960K	16x
3D	Hemisphere Cylinder	15M	19x
3D	Wing-Body-Tail (DPW4)	10M	7x
3D	Wing-Body (DPW5)	15M	< 1x

From Quarter Horse to Thoroughbred In Target CFL

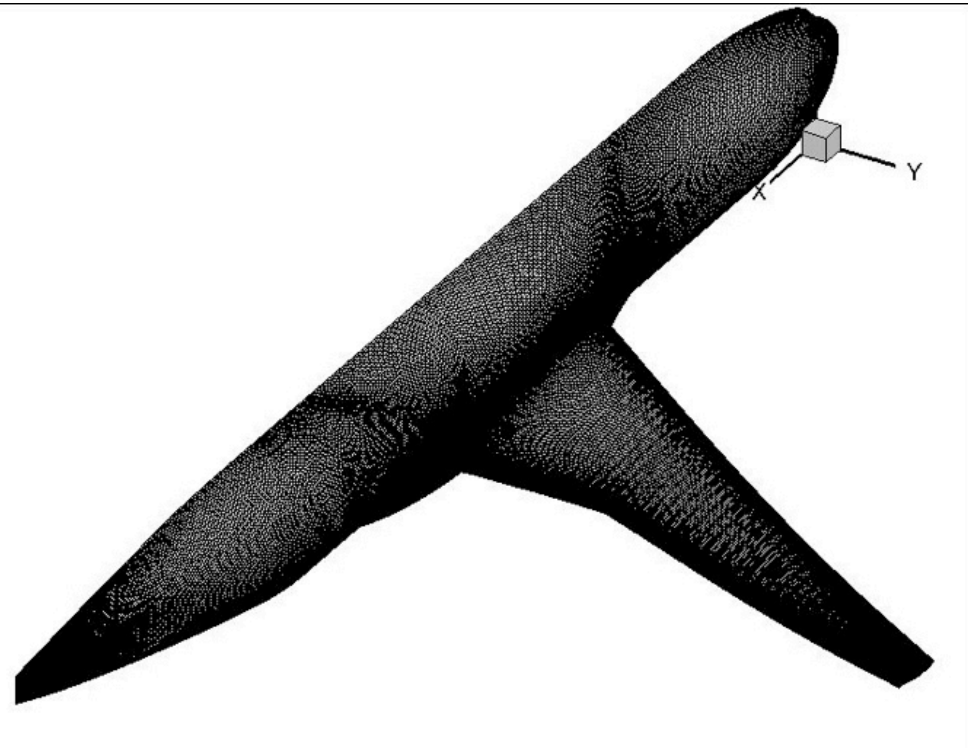
NACA 0012; M=0.15; Alpha = 15

Cycles to Machine Zero Residuals with Full Multigrid Cycle

Grid Density	Agglomerated Multigrid V(3,3) Cycles CFL=200	Structured Multigrid V(2,2) Cycles CFL=10,000
Grid 1 (Fine)	276	24
Grid 2 (Medium)	241	23
Grid 3 (Coarse)	216	24

Transonic Wing-Body

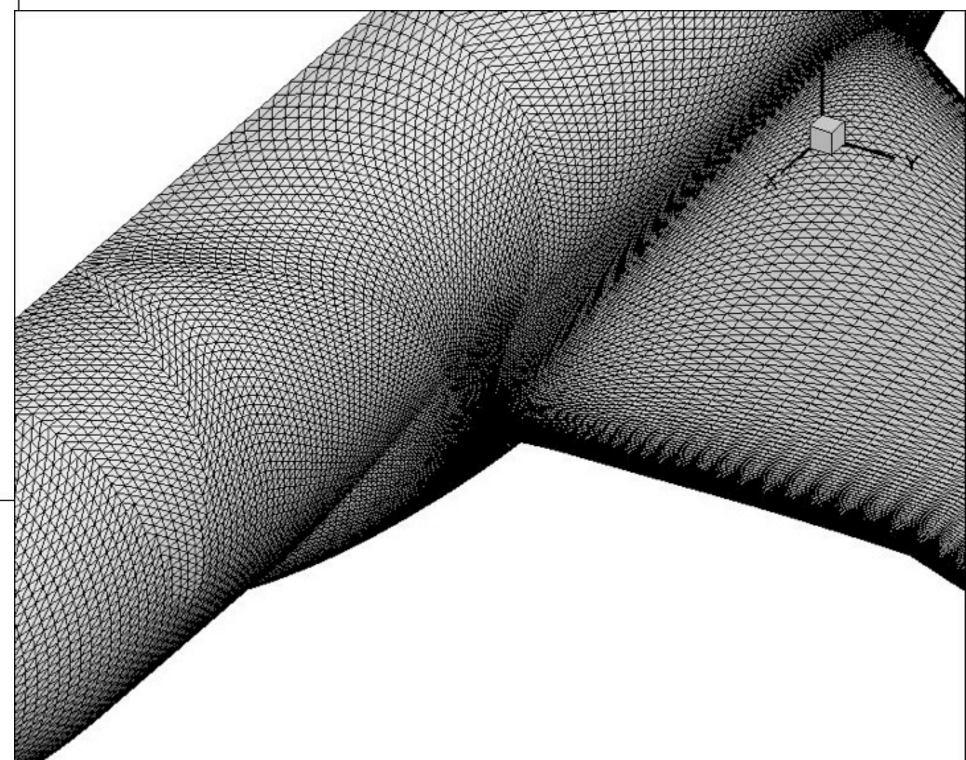
Drag Prediction Workshop 5



Hybrid unstructured grid

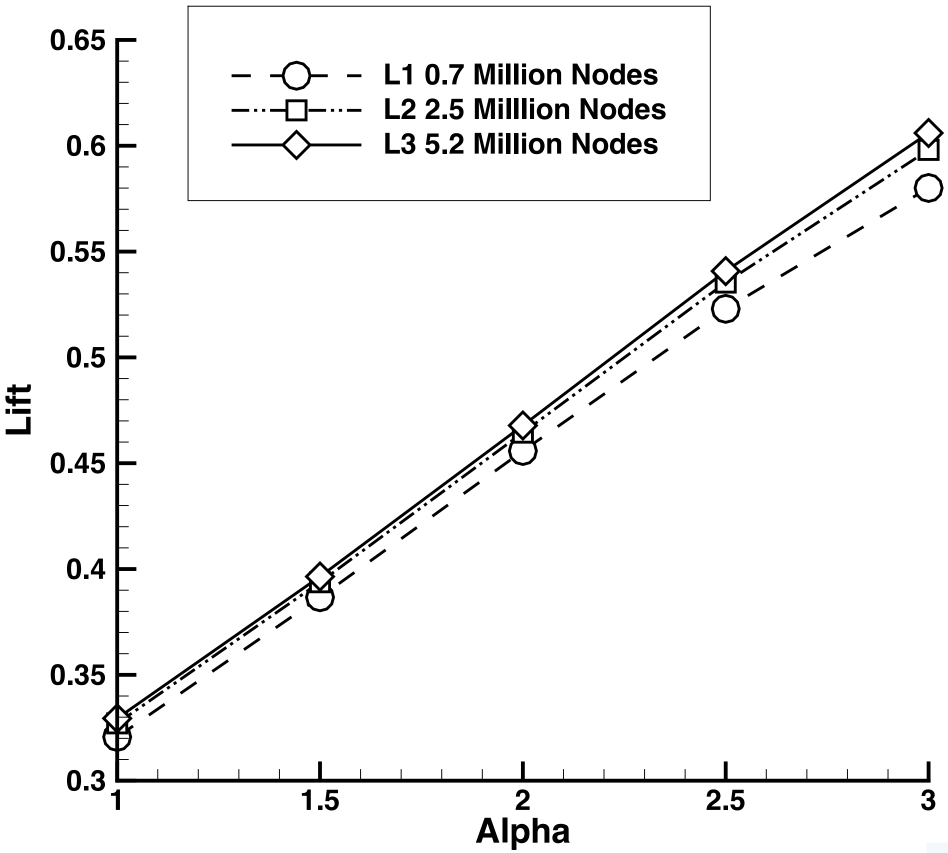
- prismatic near-field
- tetrahedral far-field

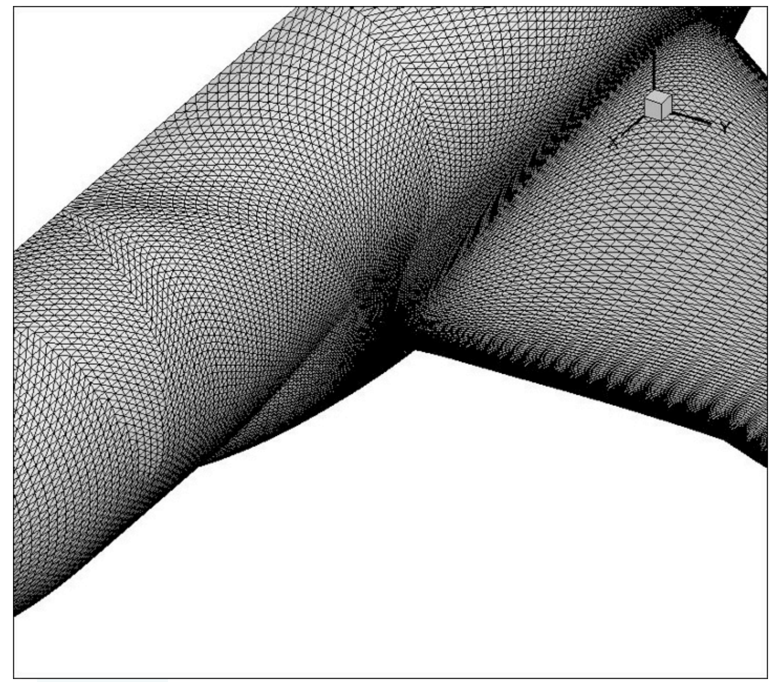
Structured Grid Designation	Nodes
L1	0.7 Million
L2	2.2 Million
L3	5.2 Million



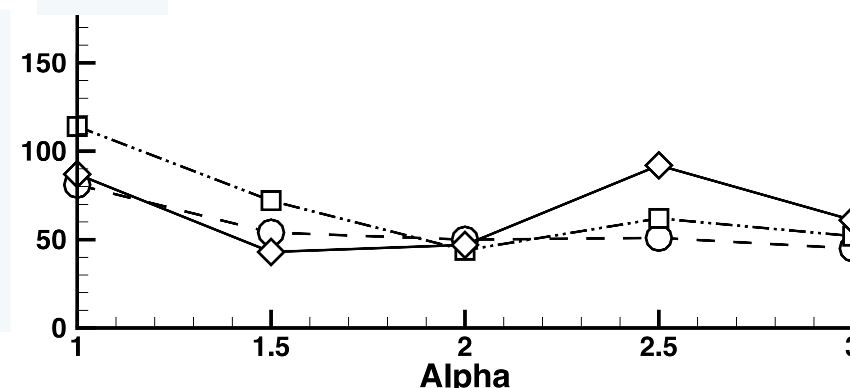
Transonic Wing-Body

Structured V(2,2) Multigrid Cycle; CFL=10,000
M=0.85 ; Alpha Continued [1 to 3]



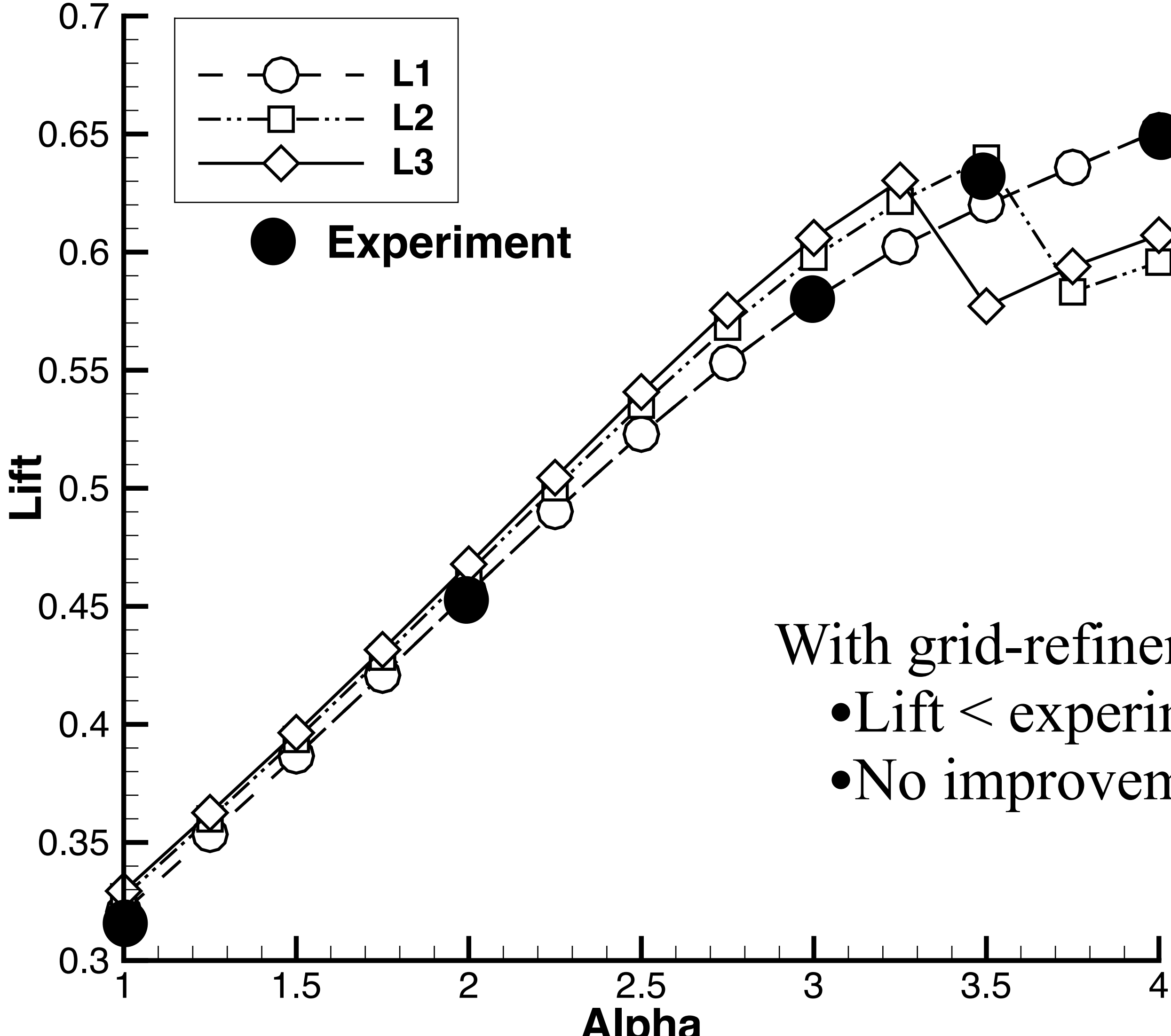


Cycles to Converge
Six Orders in Residual



Transconic Wing-Body

M=0.85 ; Alpha Continued [1 to 4]

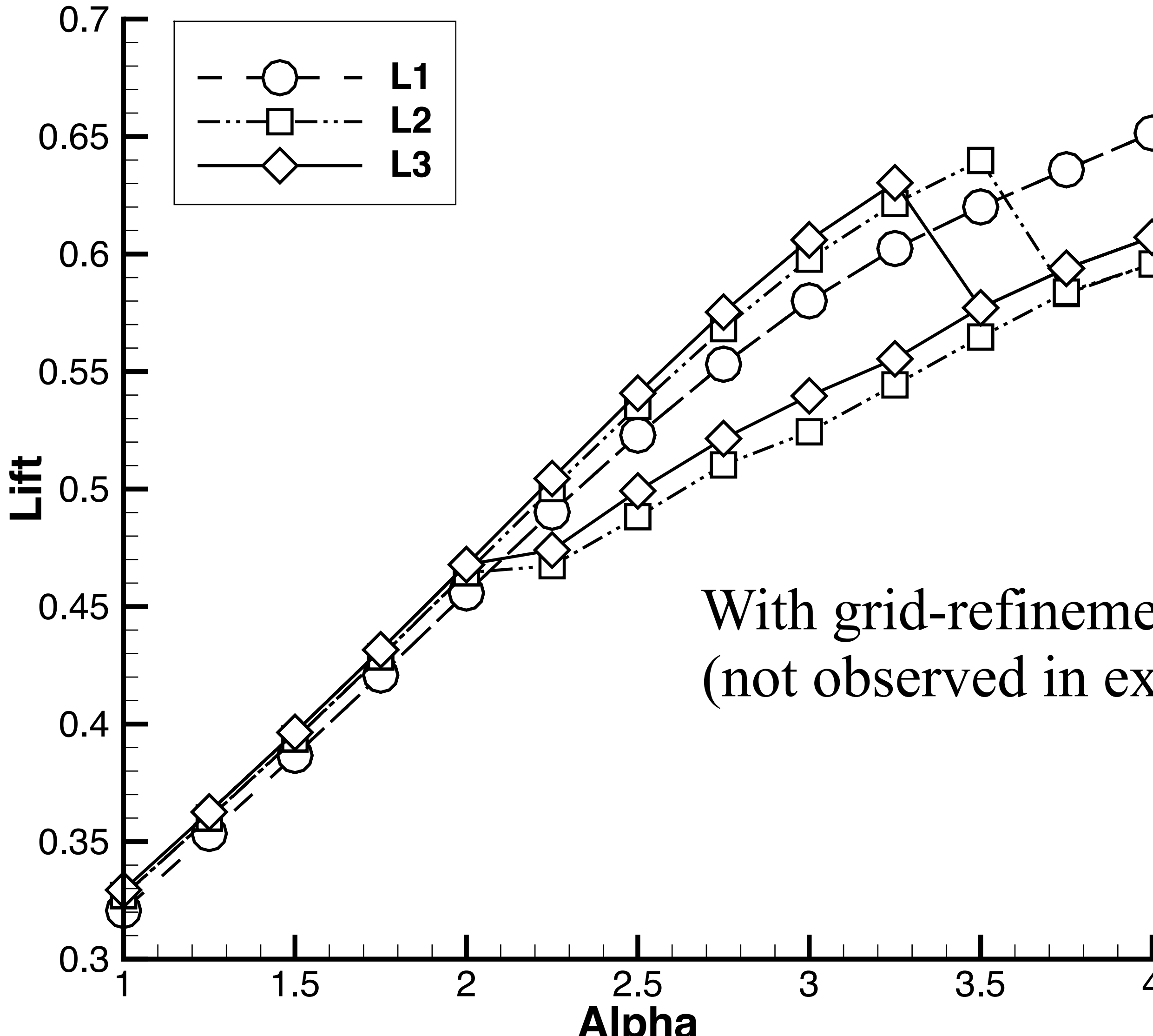


With grid-refinement for alpha > 3

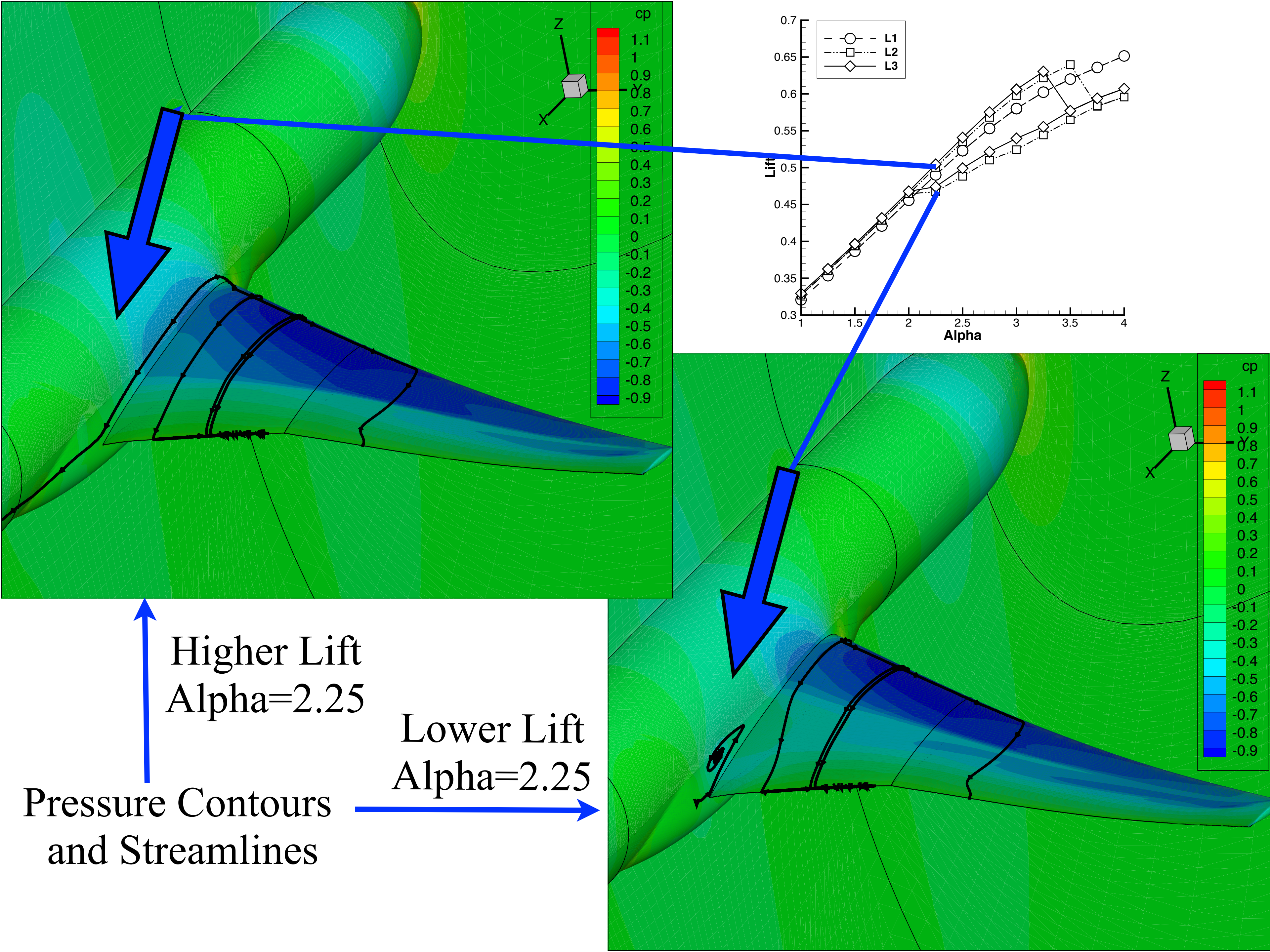
- Lift < experiment
- No improvement with multigrid

Transonic Wing-Body

M=0.85 ; Alpha Continued [1 to 4], then decreasing



With grid-refinement, strong hysteresis
(not observed in experiment)


cp
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
Z
Y
X
L1
L2
L3
Lift
Alpha
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
1
1.5
2
2.5
3
3.5
4
Higher Lift
Alpha=2.25
Lower Lift
Alpha=2.25
Pressure Contours
and Streamlines

Concluding Remarks

- Hierarchical solver with adaptive time step control proven useful for turbulent flows
  - Single grid and multigrid methods
  - Used in comparisons/adaptation of Park (Monday AM)
- Agglomeration multigrid assessed over range of tests
  - V(3,3) cycle with CFL=200
  - Comparable performance with structured-grid multigrid
  - Substantial improvement over single grid method
- Structured multigrid assessed over smaller range of tests
  - V(2,2) with CFL=10,000
  - Fast convergence for 2D and 3D comparable to inviscid

Future Research

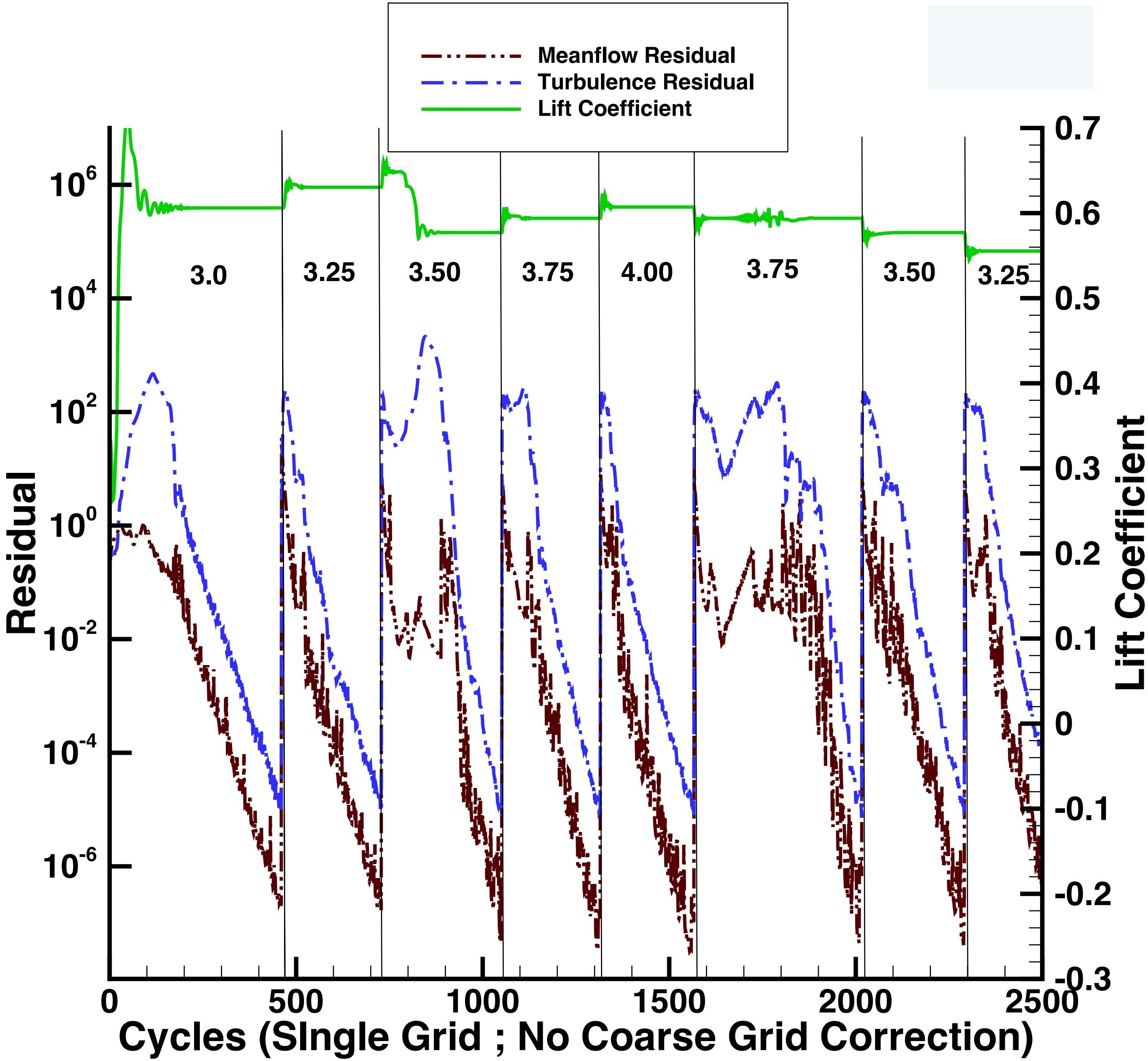
- Single grid solver
  - Refinements in adaptation strategy to reduce sensitivity to selectable parameters (robust controller)
- Multigrid
  - Assess agglomeration multigrid with higher CFL numbers
  - Understand limitations observed for DPW5 grids
  - Apply *ideal* multigrid tools to assess where further inroads are possible
    - *Ideal relaxation* (tests coarse grid correction)
    - *Ideal coarse grid* (tests relaxation)

Research Possible through the NASA Fundamental Aeronautics Program

Support of cross-cutting technology development
- Peter Coen (Supersonics)
- Mike Rogers (Subsonic Fixed Wing)
- Susan Gorton (Rotorcraft)
- Mujeeb Malik (Revolutionary Computational Aerosciences)

First two authors supported by NASA contracts
- NNL12AB00T "Improvements of Unstructured Finite-Volume Solutions for Turbulent Flows"
- NNL09AA00A "Efficient Iterative Solutions for Turbulent Flows

BackUp Slides Follow


Meanflow Residual
Turbulence Residual
Lift Coefficient
3.0
3.25
3.50
3.75
4.00
3.75
3.50
3.25
Residual
Lift Coefficient
Cycles (SIngle Grid ; No Coarse Grid Correction)

Coarse Grids : AgMG vs StMG

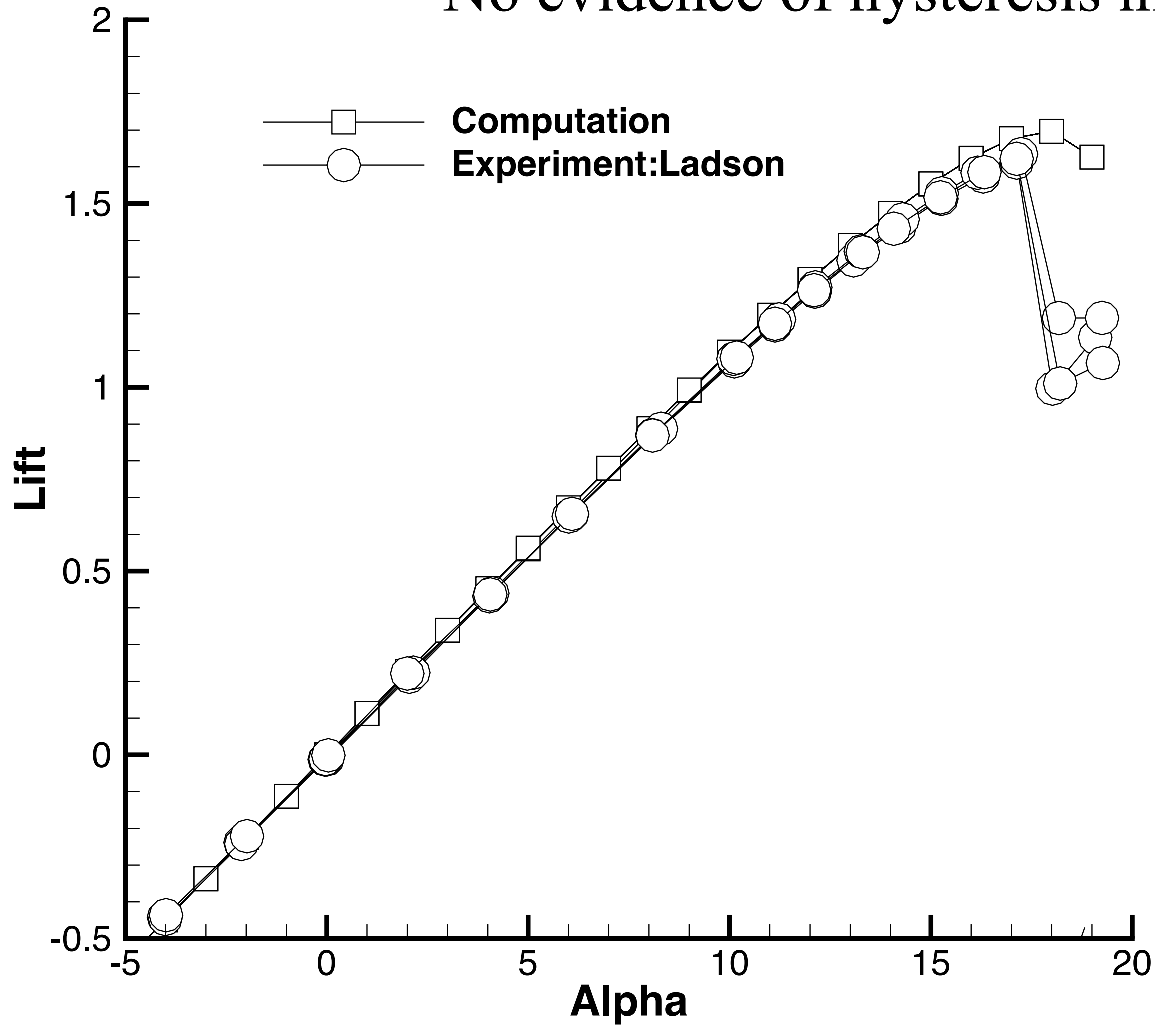
Property	Agglomerated	Structured
Coarsening	Hierarchical (3-level)	Full (all levels)
Accuracy	First Order Inviscid Second Order Viscous	Second Order Inviscid Second Order Viscous
Jacobians	Approximate Viscous	Approximate Inviscid
Restriction	Conservative with Residual Averaging	Full Weighting (prolongation transpose)
Prolongation	Linear --> Constant (viscous curved)	Linear (structured mapping)
Cycle / CFL	V(3,3) / CFL=200	V(2,2) / CFL=10,000

NACA 0012 Airfoil

M=0.15; Finest Grid (919K)

Alpha Continued From [10 to 19] then [19 to -5] then [-5 to -3]

No evidence of hysteresis in computation



NACA 0012 Airfoil

M=0.15; Finest Grid (919K)

Alpha Continued From [10 to 19] then [19 to -5] then [-5 to -3]

O(10) improvement with multigrid

O(10) more cycles near zero lift