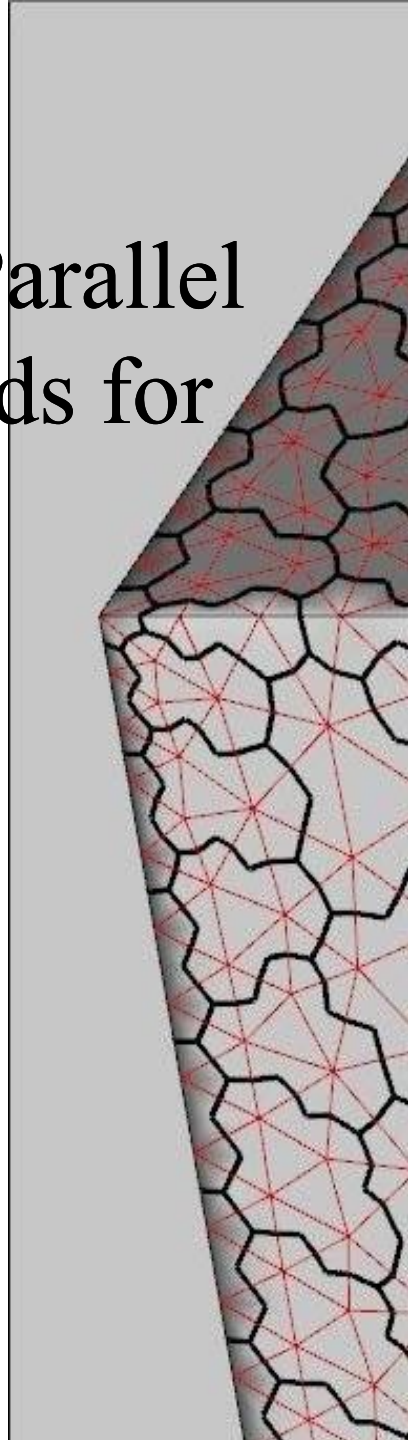
*AIAA Computational Fluid Dynamics Conference, 2011, Hawaii*

# Development and Application of Parallel Agglomerated Multigrid Methods for Complex Geometries



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*Acknowledgements:*

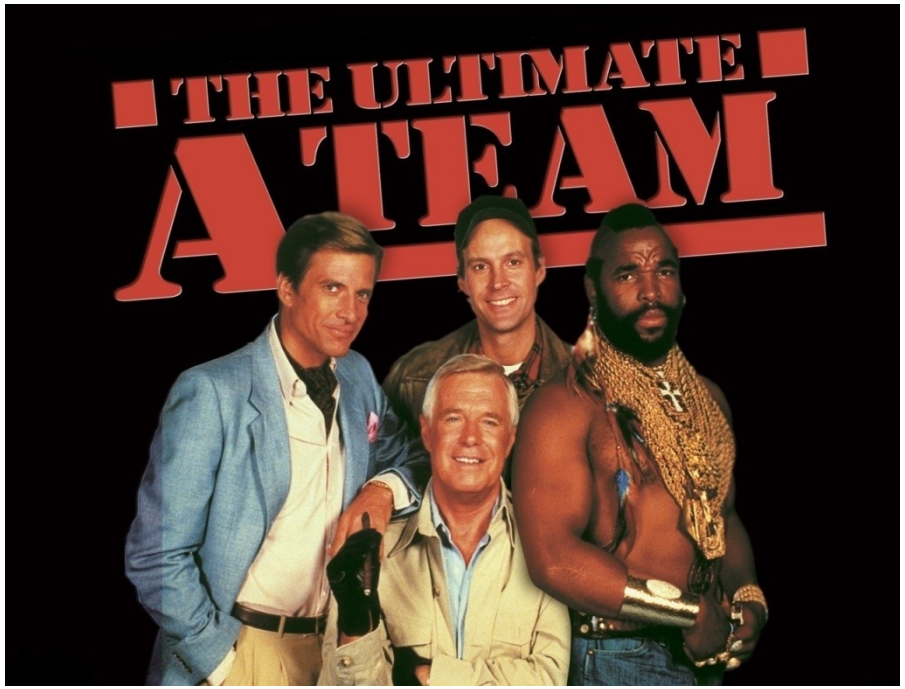
James L. Thomas,

Dana Hammond,

Bil Kleb

*NASA Langley Research Center*

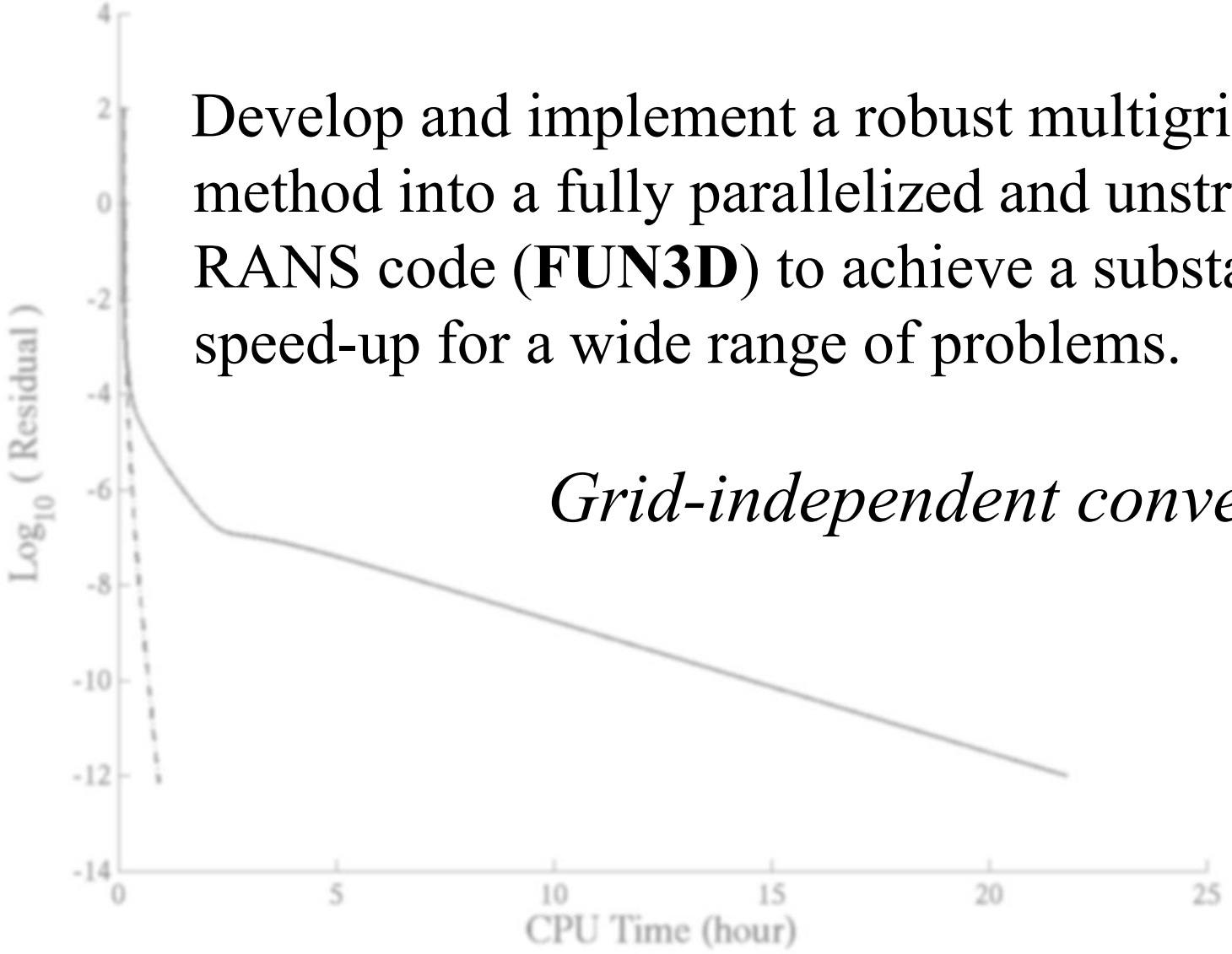
*"If you want a speed-up, if no one else can help, and if you can find them, maybe you can hire the A-team."*



# OBJECTIVE

Develop and implement a robust multigrid(MG) method into a fully parallelized and unstructured RANS code (**FUN3D**) to achieve a substantial speed-up for a wide range of problems.

*Grid-independent convergence.*



THE ULTIMATE
# A-TEAM

*Chapter 1. Eliminate LFE quickly*

*Chapter 2. Consistent with Target*

*Chapter 3. Secure Interior Connection*

*Chapter 4. Leave Everything to Face*

*Chapter 5. Twice is Nice and Smooth*

# *Chapter 1.*

# *Eliminate LFE quickly.*

# FUN3D: Single-Grid Scheme

**Residual (Node-centered FV discretization):**

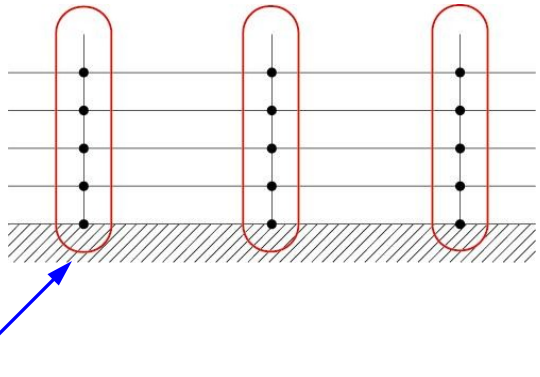
2nd-order edge-based (inviscid, Roe) + Green-Gauss (viscous).
Turbulence equation loosely-coupled with mean-flow equations.

$$\hat{R}(U) = 0$$

**Iteration (Implicit):**

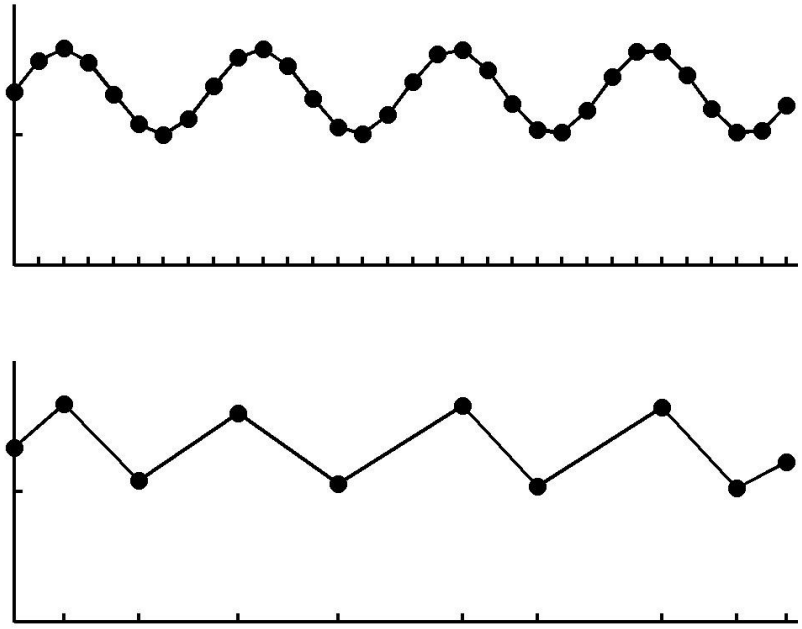
Update: $$U^{k+1} = U^k + \delta U$$

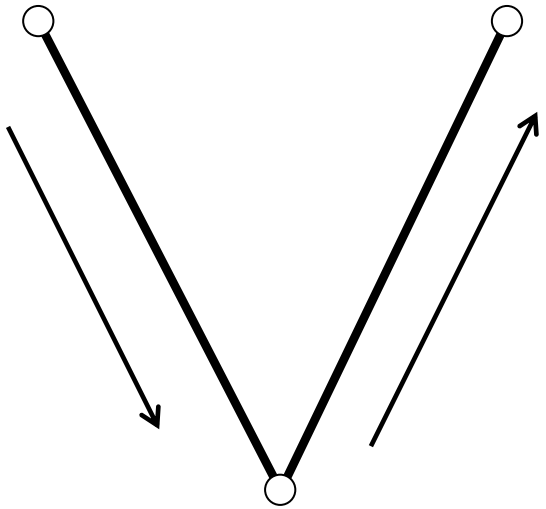
Linear Sweep: $$\left(\frac{\Omega}{\Delta\tau} + \frac{\partial \hat{R}^*}{\partial U}\right)\delta U = -\hat{R}(U)$$



Multi-color Gauss-Seidel point/**line** scheme

# Multigrid





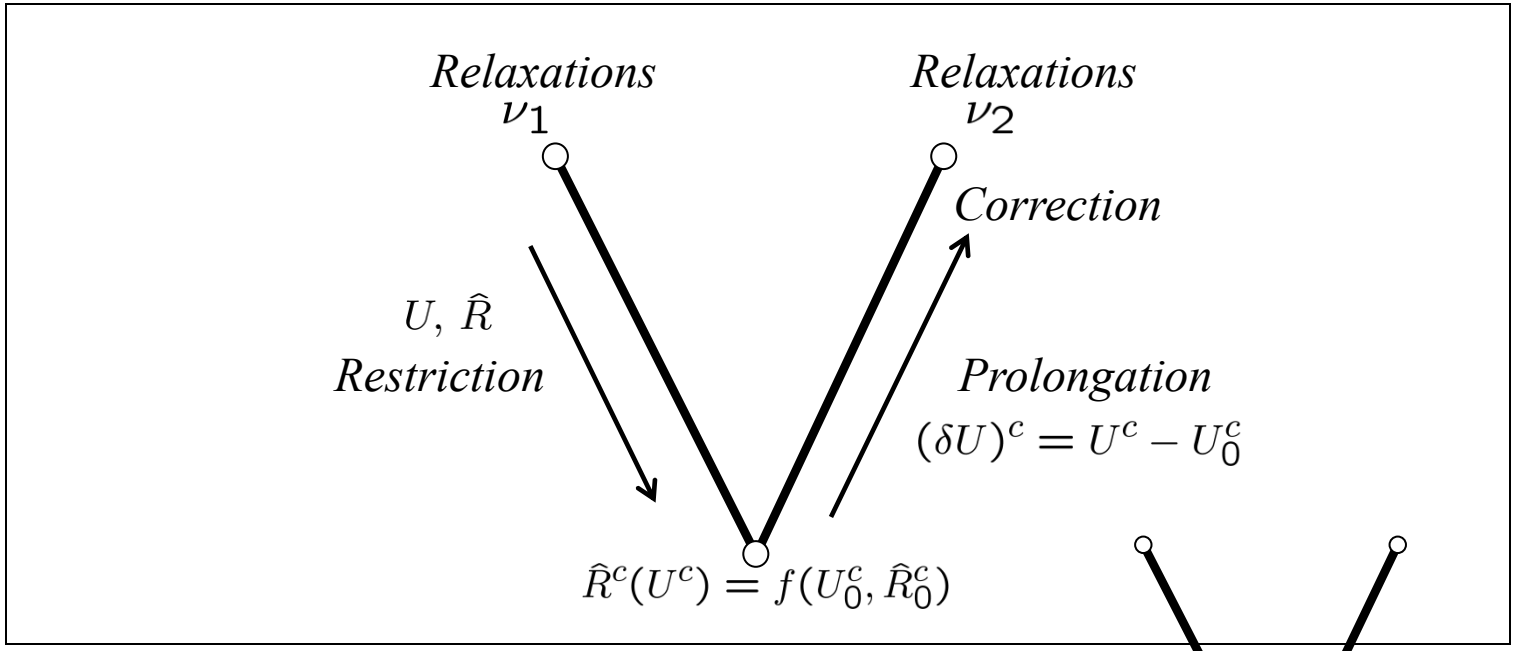
Fedorenko (1961)

Brandt (1977)

Remove low-frequency errors quickly by MG.

# Multigrid: Full-Approximation Scheme

**Two-grid $V(\nu_1, \nu_2)$ cycle - a building block:**




Apply this recursively to multiple coarse grids:

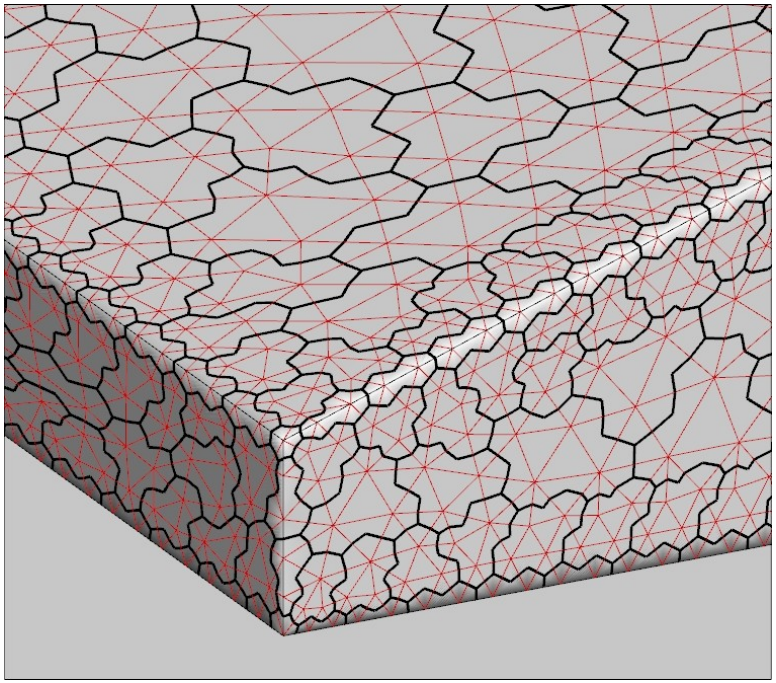
Multigrid requires a fewer number of linear sweeps and Jacobian evaluations:

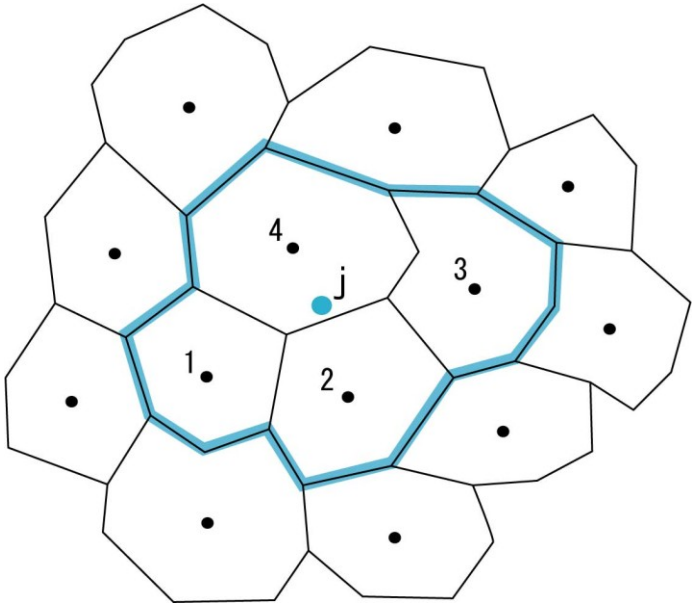
Cost[ MG V(3,3) with 5 sweeps ] = **2** × Cost[ SG with 15 sweeps ]

# Agglomerated Multigrid

**Merge control volumes**

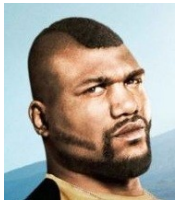
Smith(1990), Lallemand-Steve-Dervieux(1992), Mavriplis-Venkatakrishnan(1995), Sorensen-Hassan-Morgan-Weatherill(2003)…



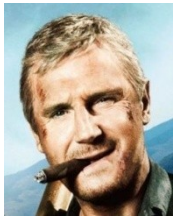



**MG convergence can deteriorate on fine viscous grids…**

Venkatakrishnan(1996)



*B.A.:*
*“It slows down for diffusion on fine grids!”*



*Hannibal:*
*“We knew it. We just have to do it right.“*

*Continues…*

# *Chapter 2.*

# *Consistent with Target*

# Viscous Coarse-Grid Discretization

( Inviscid**:** First-order version of the fine-grid scheme. )

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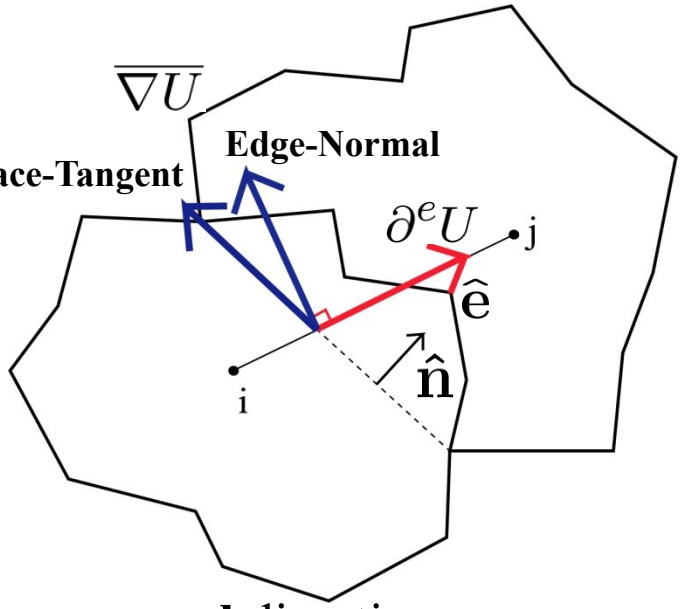
**How to construct the gradient at face:**




*Edge-Term:* $\partial^e U = \dfrac{U_j - U_i}{\Delta l_{ij}}$

Average-LSQ gradients: $\overline{\nabla U} = \frac{1}{2}(\nabla U_i^{LSQ} + \nabla U_j^{LSQ})$

- ***Consistent schemes:***

  ***Edge-normal*** Avg-LSQ scheme: $\overline{\nabla U}$ in **edge-normal** direction

  ***Face-tangent*** Avg-LSQ scheme: $\overline{\nabla U}$ in **face-tangent** direction

- ***Inconsistent schemes:***

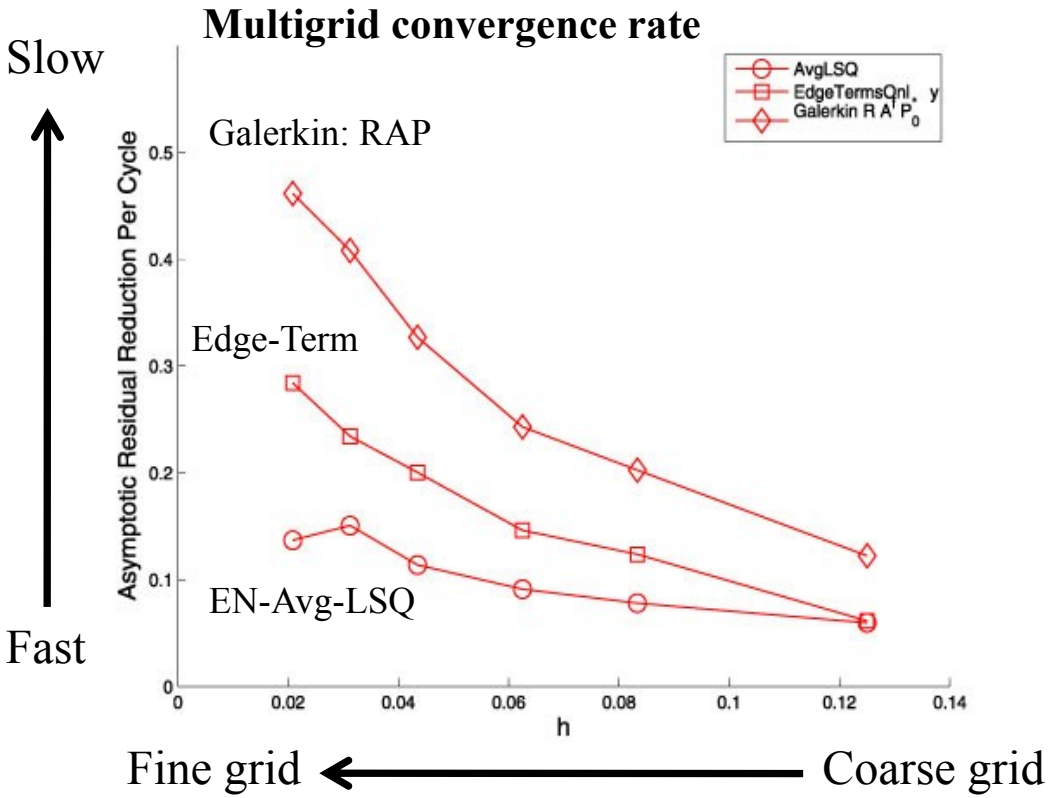
  Edge-term-only scheme: Take $\overline{\nabla U} \to 0$ in the Avg-LSQ scheme.

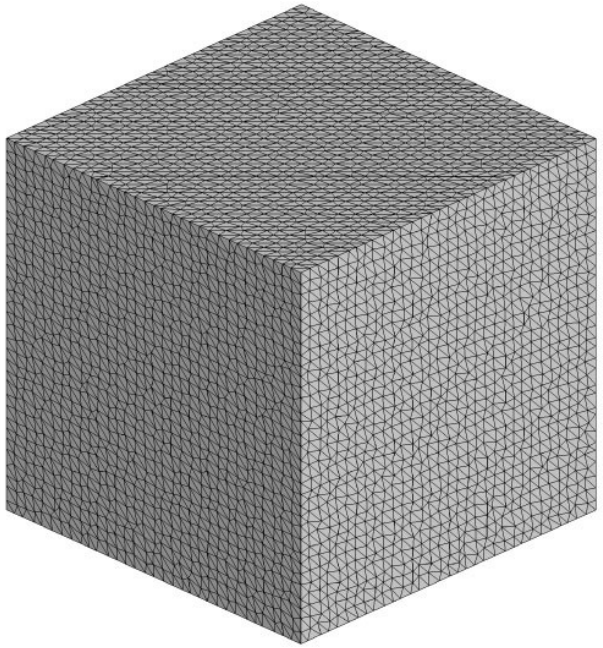
  Scaled Galerkin Construction: $A^c = RA^f P$

**[Thomas, et. al. C&F2011]**

# Deterioration on Fine Grids

Model diffusion on Isotropic tetrahedral grids (AIAA Journal 2010)

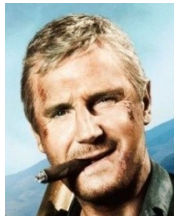


*Inconsistent schemes deteriorate the multigrid convergence.*

Similarly for stretched mixed grids (C&F2011)

*Face:*
*"Gradient reconstruction is failing!"*



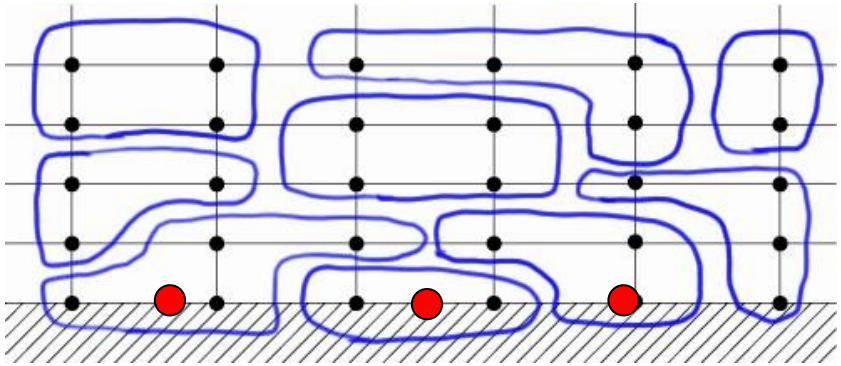
*Hannibal:*
*"Looks like stencils are degenerated.*
*We must do something about it."*

*Continues…*

# *Chapter 3.*

# *Secure interior connections.*

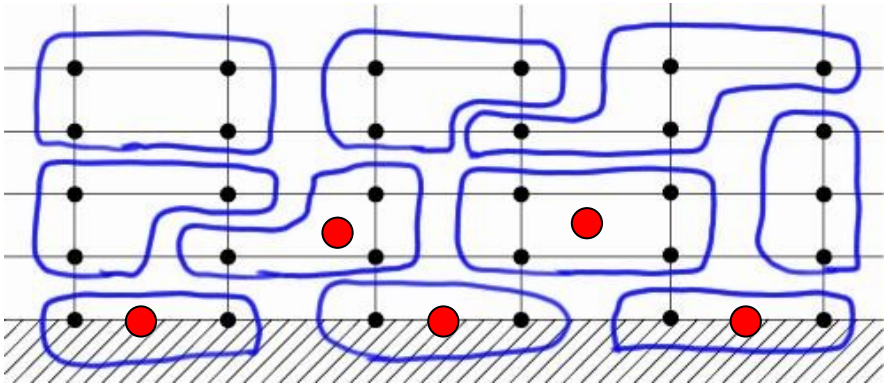
# Interior Connection Problem



**Interior connection is lost…**
**Degenerated LSQ stencil…**

How about not agglomerate boundary nodes with interior nodes?

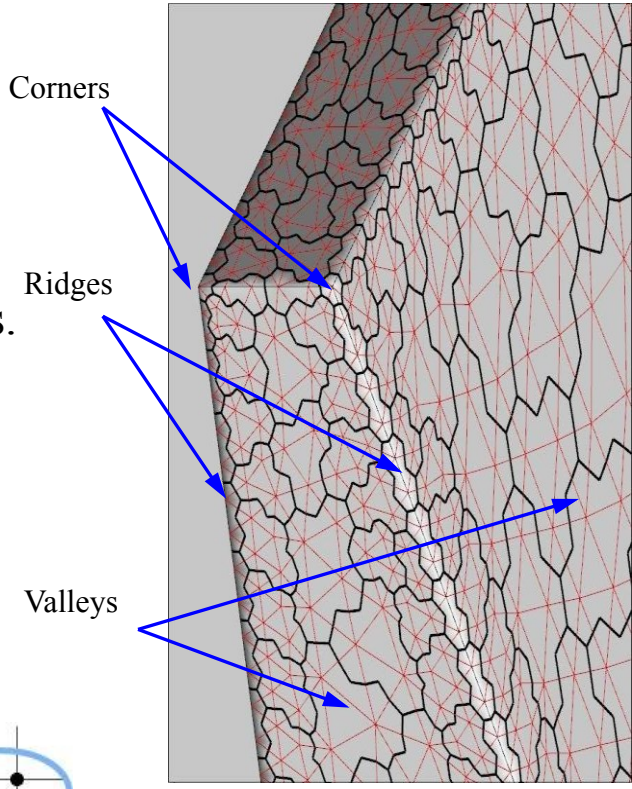
**Interior connection is guaranteed.**
**Valid LSQ stencil.**



# Hierarchical Agglomeration

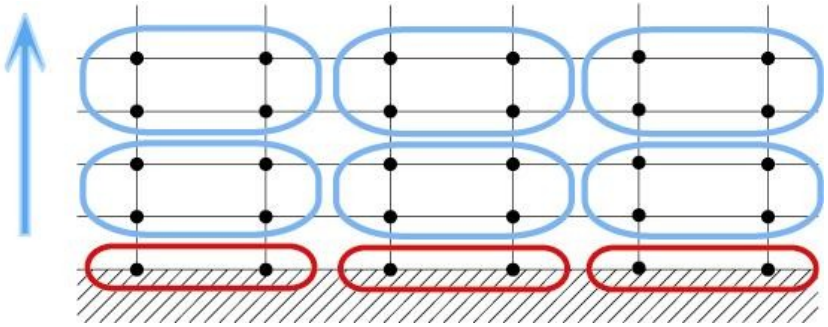
**Hierarchical Agglomeration (base scheme):**

- **Assign hierarchies**: corners, ridges, valleys, interiors.
- **Agglomerate within the topology** (for interior connection)
- **Advancing-front**: corners → ridges → valleys → interiors.




**Agglomeration Schedule:**

1. Viscous boundaries (bottom of implicit-lines)
2. Prismatic layers (implicit-line agglomeration).
3. Rest of the boundaries.
4. Interior.



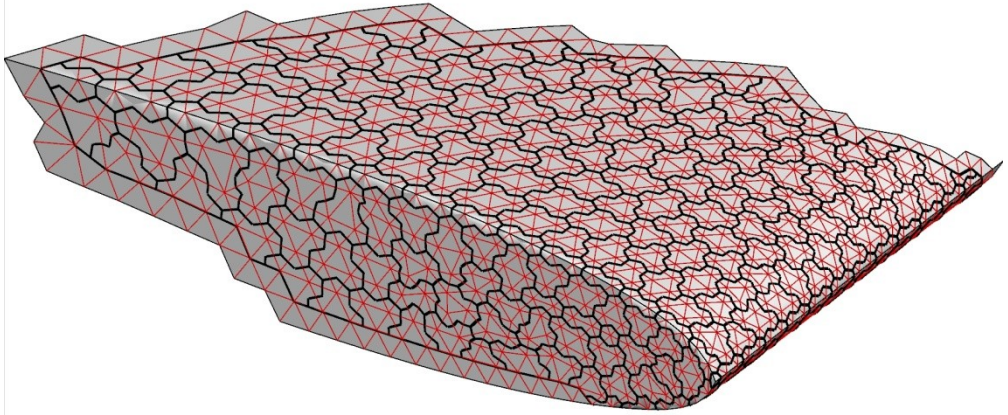
**Full-coarsening line-agglomeration (unique feature)**

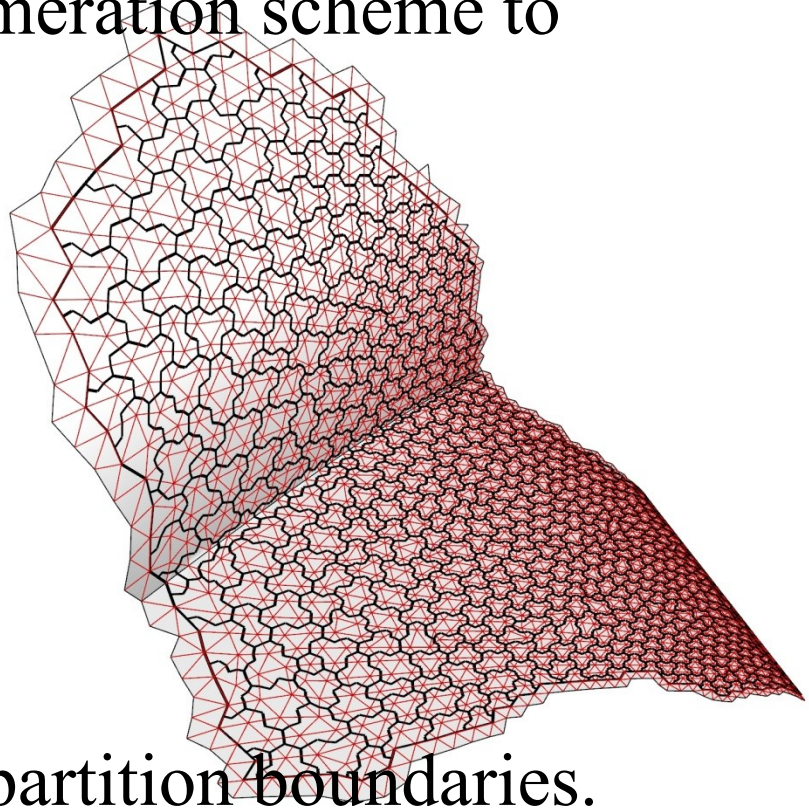
**New development in this paper.**

# Parallel Agglomeration

Partitioning does not break implicit-lines.

Apply the hierarchical agglomeration scheme to each partition independently.



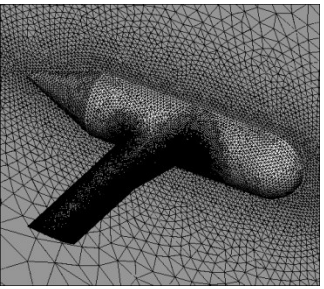


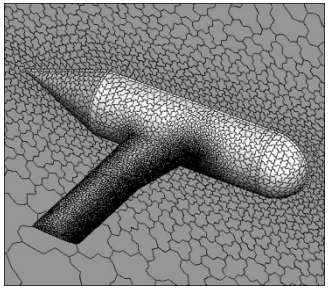
- No agglomeration across partition boundaries.
- Agglomeration is partition-dependent.

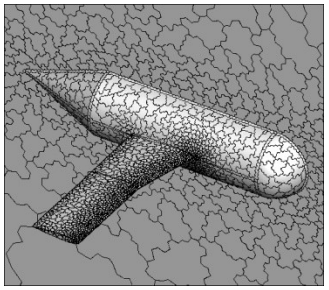
(No significant differences observed in the results.)

**New results with parallel version.**

# Inviscid: Wing-Body (1M nodes)



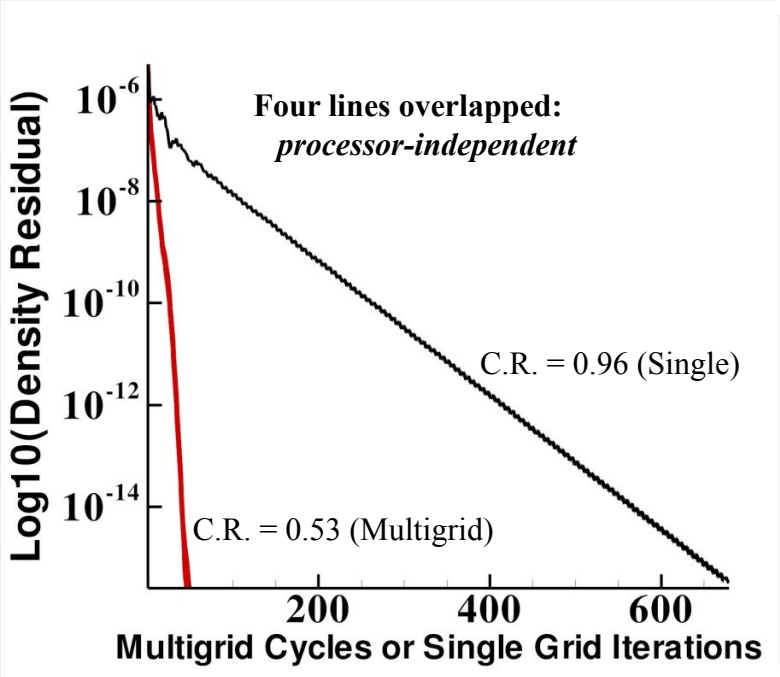


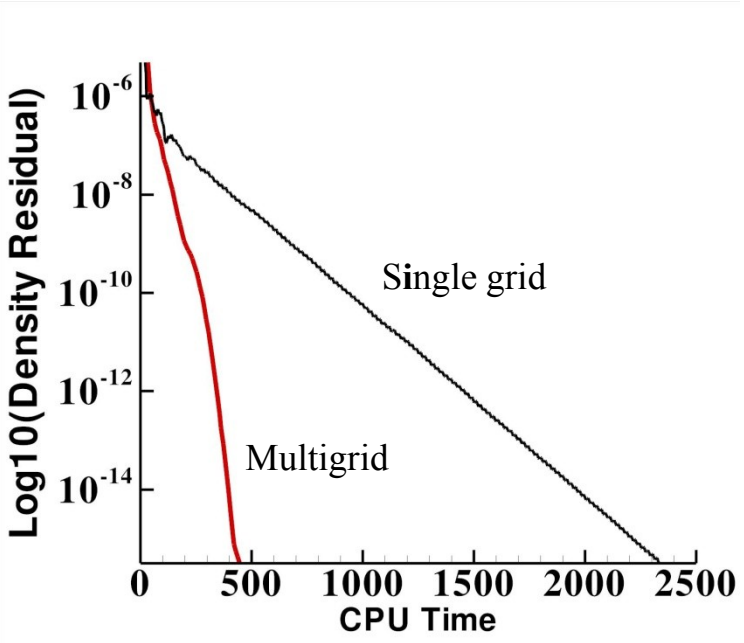


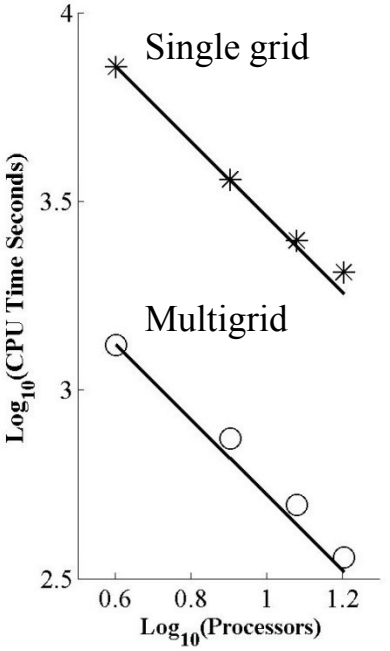
AOA=0 degree, Mach=0.3, CFL=200

Multigrid: 3-level V(2,1), 5 linear sweeps
Single Grid: 15 linear sweeps

Processors: 4, 8, 12, 16



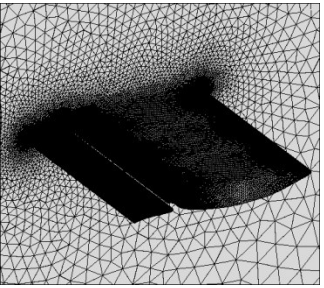



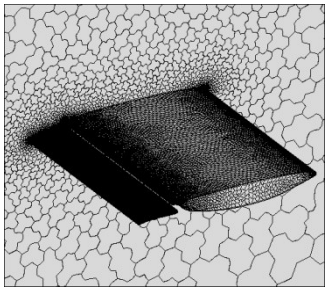

**5** times faster than the SG scheme (np=16).

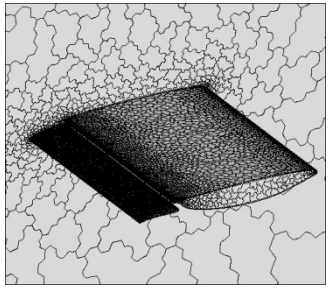
**New results with parallel version.**

# Inviscid:Wing-Flap (1.2M nodes)

AOA=2 degree, Mach=0.3, CFL=200

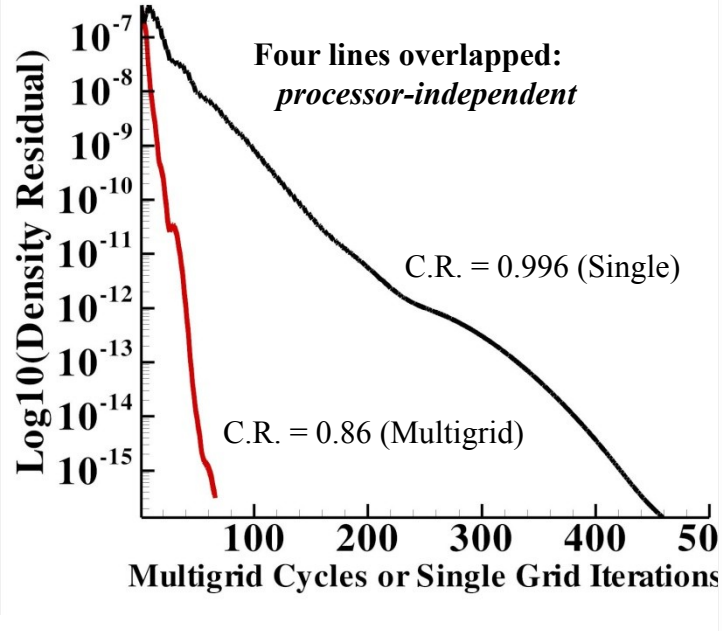


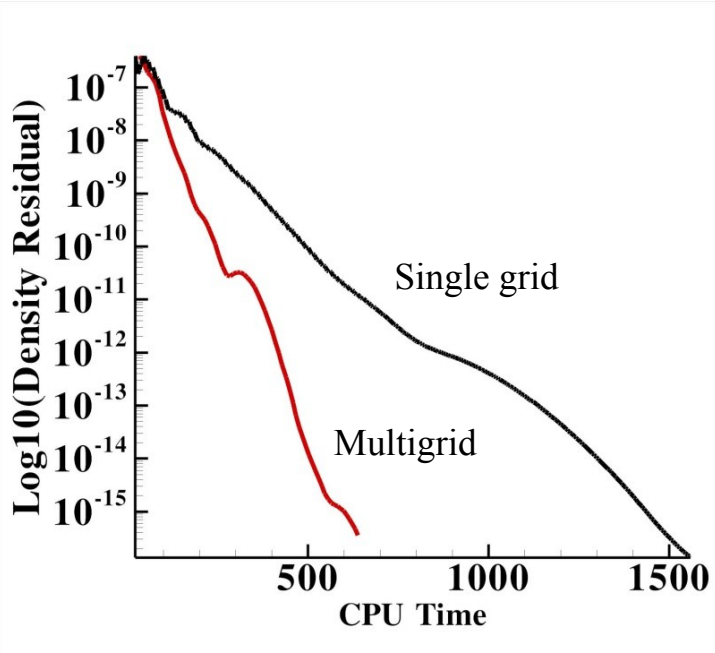


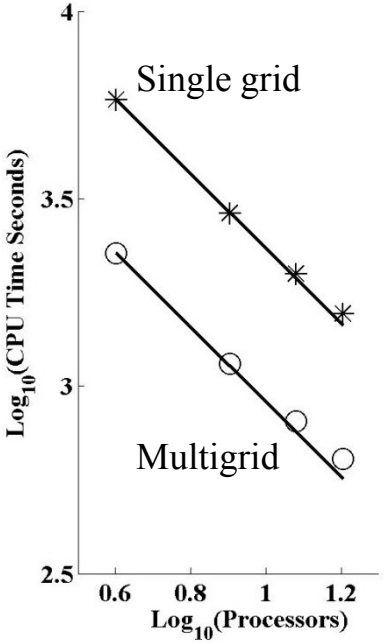


Multigrid: 3-level V(2,1), 5 linear sweeps
Single Grid: 15 linear sweeps

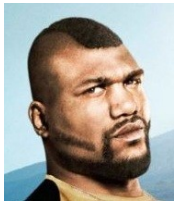
Processors: 4, 8, 12, 16



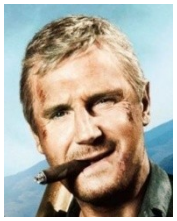




**2** times faster than the SG scheme (np=16).



*B.A.:*
*“I keep getting NaN for 3D viscous cases.”*



*Hannibal:*
*“Anybody has an idea?”*

*Face:*
*“I do.”*

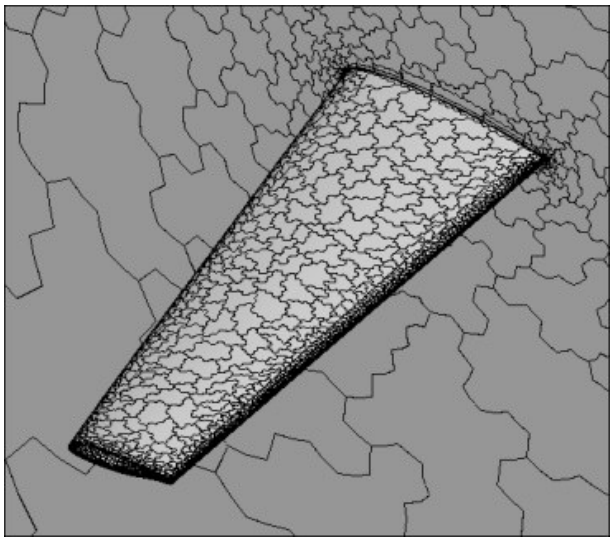
*Continues…*

# *Chapter 4.*

# *Leave Everything to Face*

# The Only Scheme That Works

**[AIAA2010]**



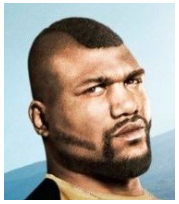
Highly skewed coarse grids.

## *Face-Tangent Avg-LSQ Scheme*

Other schemes failed…. (MG blows up…)

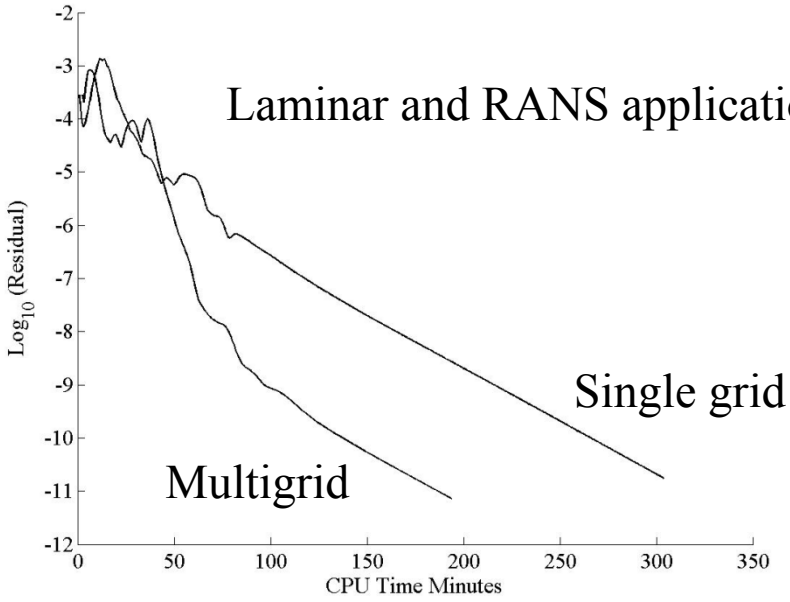
$$\nabla U = \overline{\nabla U} + \frac{1}{\hat{\mathbf{n}} \cdot \hat{\mathbf{e}}} \left[ \partial^e U - \overline{\nabla U} \cdot \hat{\mathbf{e}} \right] \hat{\mathbf{n}}$$

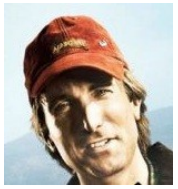
**Large edge-term contribution** **[Thomas, et. al. C&F2011]**



*B.A.:*

*"Hey, multigrid is slowing down!"*



*Murdock:*

*"Don't worry. We just need to push hard!"*

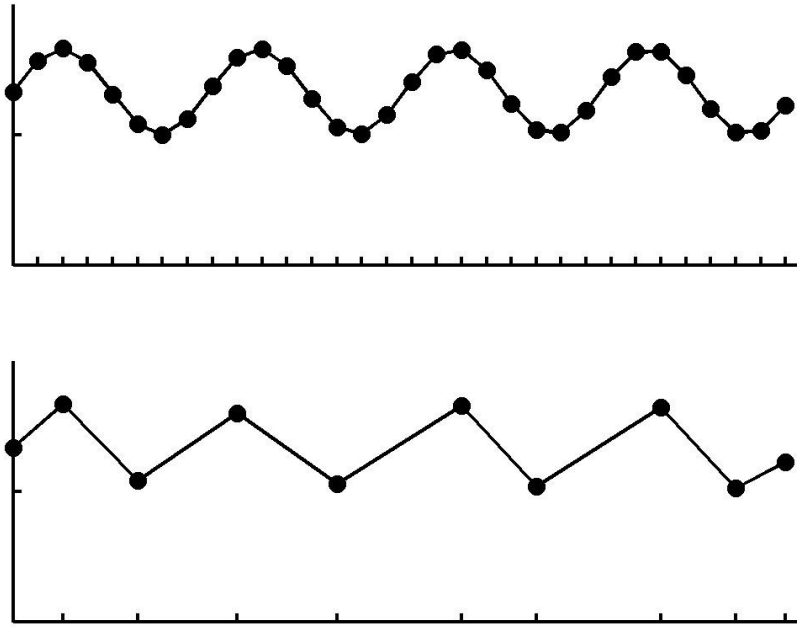
*Continues…*

# *Chapter 5.*

# *Twice is Nice and Smooth*

**New development in this paper.**

# Smooth Enough before Go Down



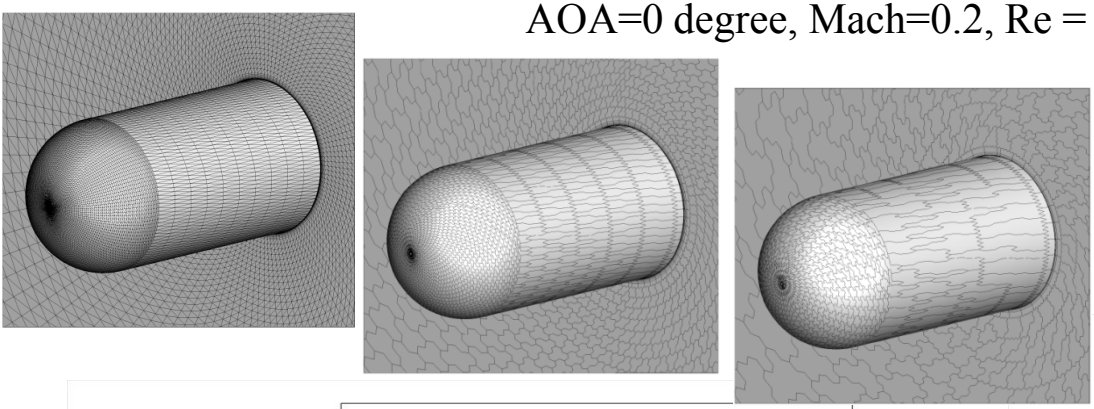
*Apply V(3,3) instead of V(2,1)*

*More expensive, but it pays off.*

**New results with parallel version.**

# Laminar: Hemisphere Cylinder

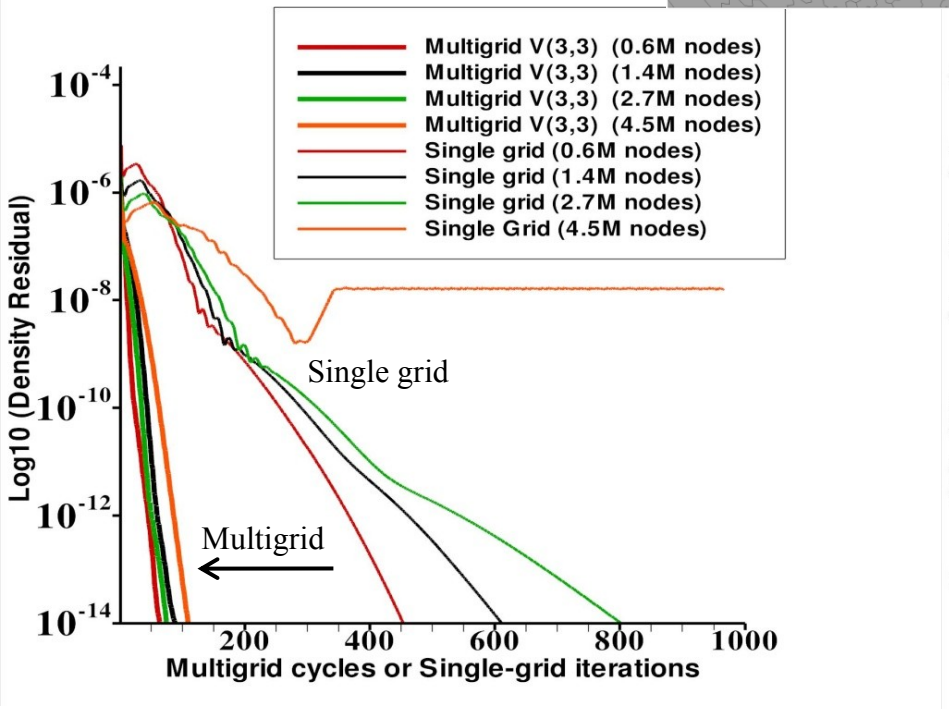
AOA=0 degree, Mach=0.2, Re = 400, CFL=200

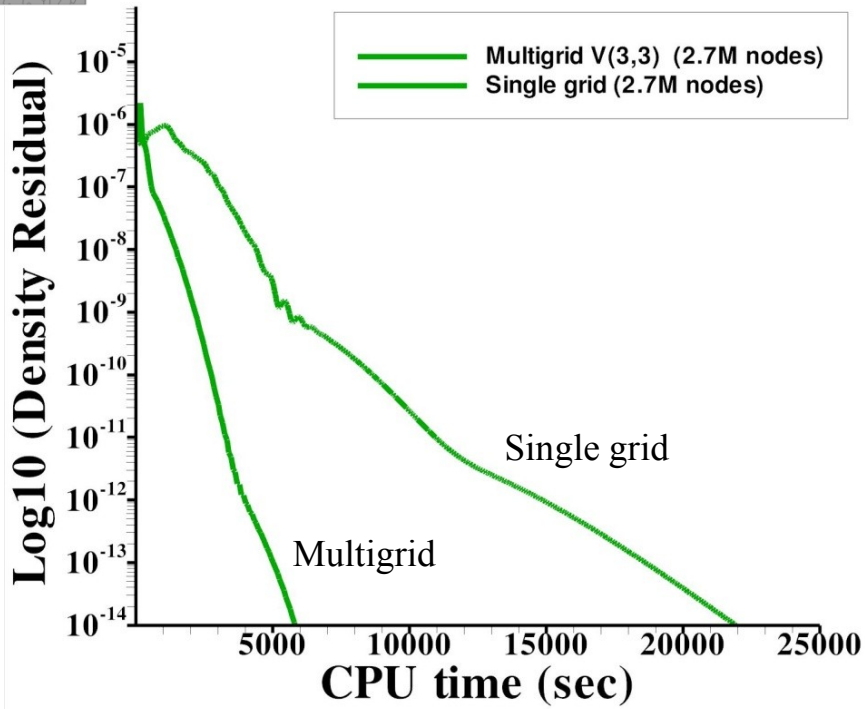


Multigrid: 3-level V(3,3), 10 linear sweeps
Single Grid: 25 linear sweeps

4 Grids: 0.6, 1.4, 2.7, 4.5 million nodes.
(tetra-prism mixed grids)

16 processors

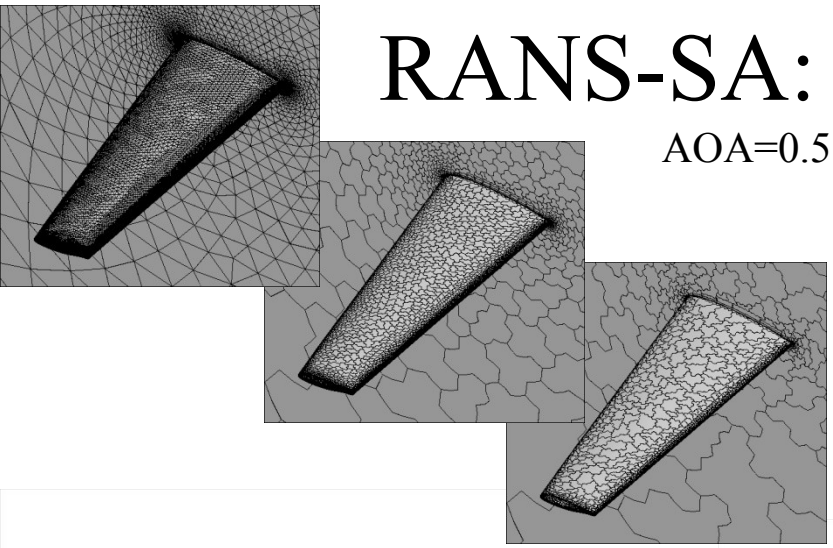



SG slows down for finer grids: 0.94, 0.97, 0.98, 1.0
MG does not slow down: 0.79, 0.85, 0.83, 0.82

MG is nearly 4 times faster for 2.7M grid.

**New results with parallel version.**

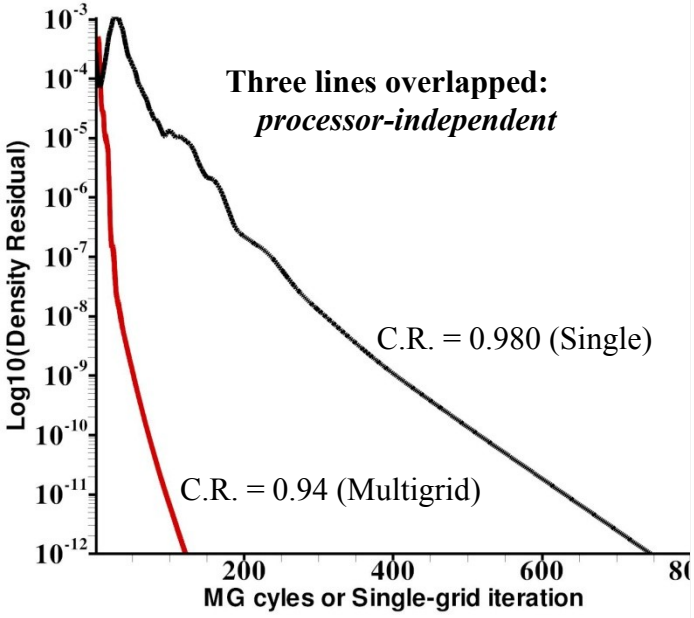


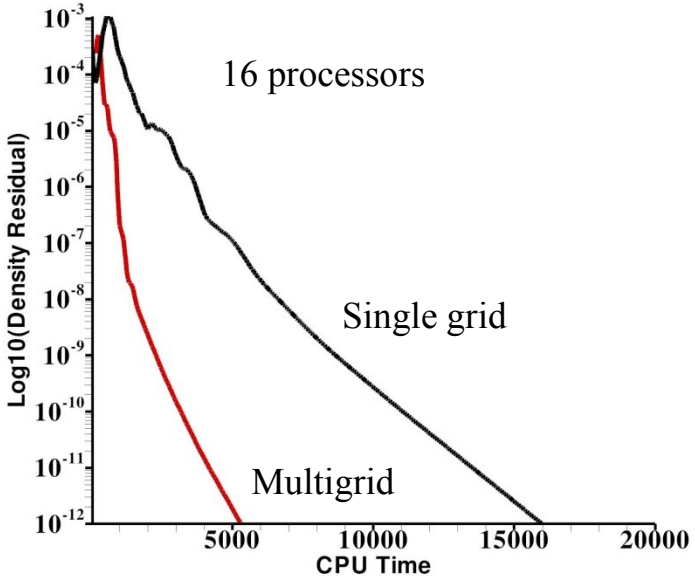
# RANS-SA: DPW-W2 (1.9M nodes)

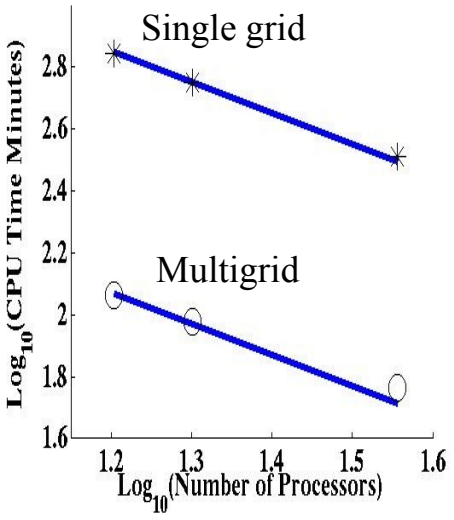
AOA=0.5 degrees, Mach=0.76, Re = 5M

Multigrid: 3-level V(3,3), 5/5 linear sweeps
Single Grid: 15/10 linear sweeps

1 Grid: 1.9 million nodes. (tetra-prism mixed grids)
Processors: 16, 20, 36



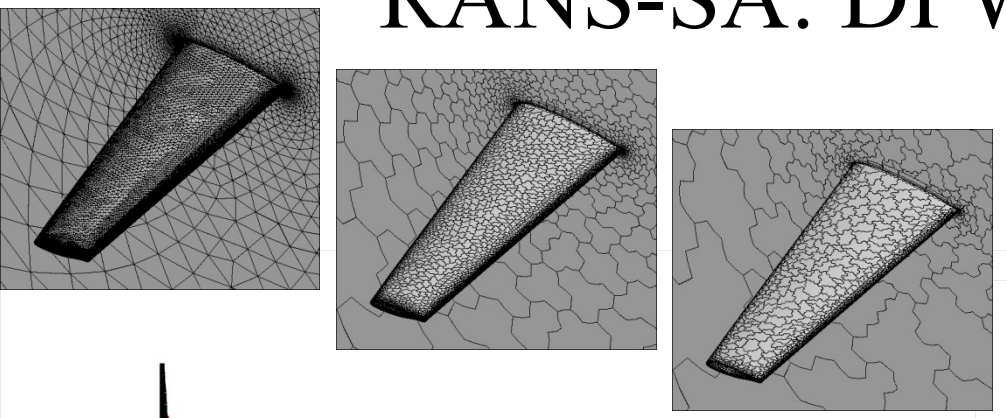



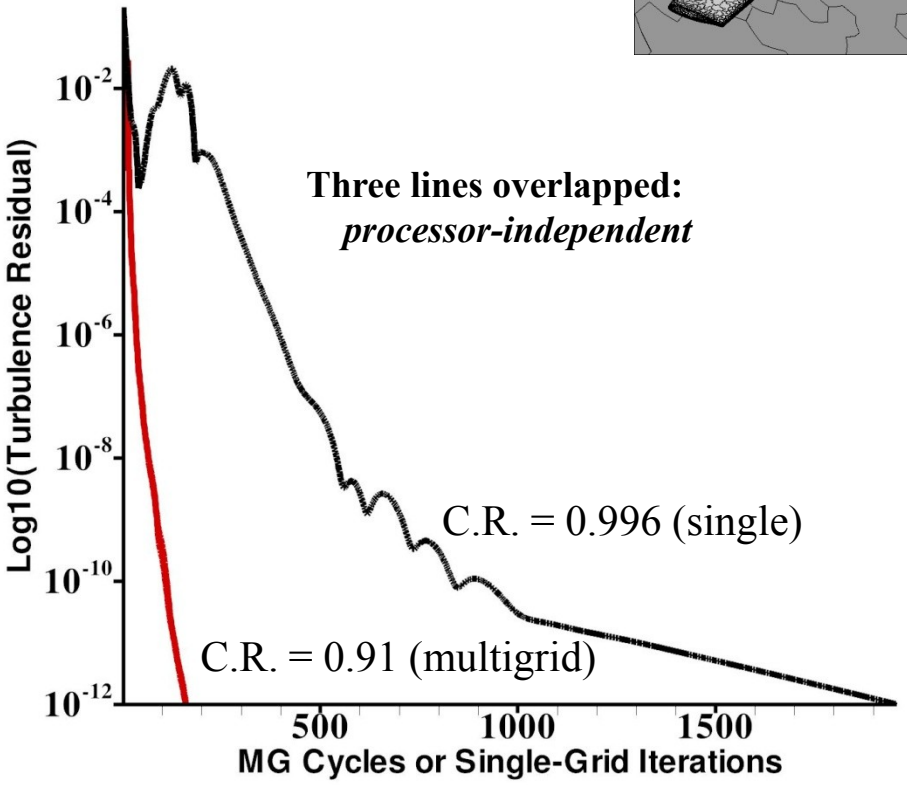

**3** times faster than the SG scheme?

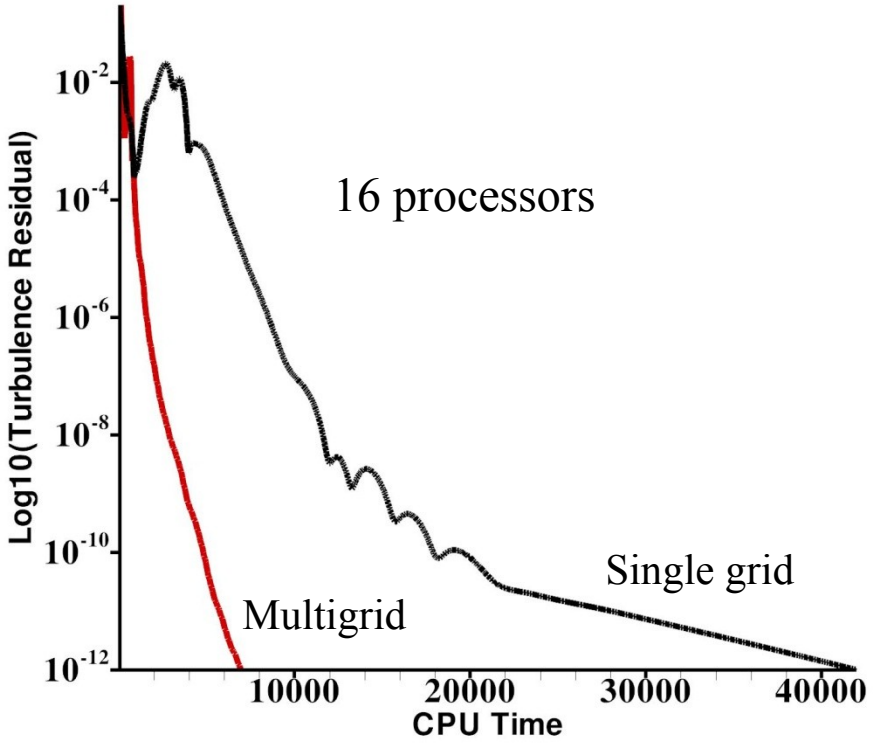
**New results with parallel version.**

# RANS-SA: DPW-W2 (1.9M nodes)



Convergence of turbulence residual.




Nearly 7 times faster than the SG scheme for turbulence.

# Conclusions and Future Work

## *Previous Developments:*

- **Consistent and robust** coarse-grid viscous scheme:

  *Face-Tangent Avg-LSQ*

- Hierarchical **full-coarsening** line-agglomeration with **line relaxation**.
- Economical V-cycle with **fewer linear-sweeps and Jacobian evaluations**

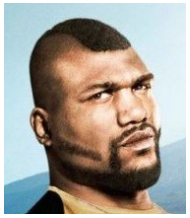
## *New Developments:*

**Agglomerated multigrid fully parallelized.**
**V(3,3)** for robust viscous computations (still economical).

## *Future Work:*

Larger-scale problems, more complex geometries, unsteady....

*Face:*
*“Come on, B.A.!  Hurry up!”*



*B.A.:*
*“I got a couple of bugs to eliminate.“*

We must complete the mission.
**THERE IS NO PLAN B.**

THE ULTIMATE
A-TEAM

*Continues to the final chapter…*