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A DUAL-MESH HYBRID LES/RANS FRAMEWORK WITH IMPLICIT CONSISTENCY

Heng Xiao and Patrick Jenny*
Institute of Fluid Dynamics, Zürich, Switzerland

INTRODUCTION

The high computational cost of Large Eddy Simulation (LES) for wall-bounded flows at high Reynolds numbers still is a major hurdle for applications to practical flows in industry and nature [3]. To overcome this difficulty, various hybrid LES/RANS methods have been proposed, of which the most widely used one is Detached Eddy Simulation (DES).

Recently, Xiao and Jenny [4] developed a novel hybrid LES/RANS framework, where LES and RANS simulations are conducted simultaneously on the entire domain on separate meshes. Relaxation forces are applied on the respective equations to ensure consistency between the two solutions. Numerical simulations were conducted based on this algorithm; the results demonstrate that the proposed method leads to satisfactory results on relatively coarse meshes, which is promising for industrial flow simulations [4,5].

In this work, to combine the consistency feature of the original formulation from the hybrid LES/RANS framework [4] and the forcing strategy used by other hybrid methods, we propose an implicitly consistent formulation coupling with a modified forcing strategy. In this modified formulation the turbulent stresses are directly corrected, which allows for a direct use of the RANS Reynolds stresses in the LES. This direct approach is advantageous especially when a sophisticated RANS model is employed.

CONSISTENT DUAL-MESH HYBRID FRAMEWORK

For simplicity, we consider incompressible flows with constant density. The momentum and pressure equations for the filtered and the Reynolds-averaged quantities can be written in a unified form as follows:

$$\frac{\partial U_i^*}{\partial t} + \frac{\partial \left(U_i^* U_j^*\right)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x_i} + \nu \frac{\partial^2 U_i^*}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}^*}{\partial x_j} + Q_i^* \tag{1a}$$

$$\text{and} \quad \frac{1}{\rho}\frac{\partial^2 p^*}{\partial x_i \partial x_i} = -\frac{\partial^2}{\partial x_i \partial x_j}\left(U_i^* U_j^* + \tau_{ij}^*\right) + \frac{\partial Q_i^*}{\partial x_i}, \tag{1b}$$

where t and x_i are time and space coordinates, respectively; ν is the kinematic viscosity, ρ is the constant fluid density, and p^* is the pressure. In the filtered equations, U_i^*, p^*, and τ_{ij}^* represent filtered velocity $\overline{U}_i$, filtered pressure $\overline{p}$, and residual stresses τ_{ij}^{sgs}, respectively. In Reynolds-averaged equations, U_i^*, p^*, and τ_{ij}^* represent Reynolds-averaged velocity $\langle U_i \rangle$, Reynolds-averaged pressure $\langle p \rangle$, and Reynolds stresses τ_{ij}, respectively. The source term Q_i^* represents the drift forces applied in the filtered equations (Q_i^L) and in the Reynolds-averaged equations (Q_i^R) to ensure consistency between the two solutions. This term will be detailed in Eqs. (3) and (4).

In this hybrid framework, the filtered equations and the Reynolds-averaged equations are solved simultaneously in the entire domain but on separate meshes. This leads to some redundancy, and the consistency is enforced with Q_i^*.

We first introduce Exponentially Weighted Average (EWA) LES quantities, in the case of the EWA velocity, it is defined as

$$\langle \overline{U}_i \rangle^{\text{AVG}}(t) = \frac{1}{T}\int_{-\infty}^{t} \overline{U}_i(t') e^{-(t-t')/T} dt', \tag{2}$$

respectively, where T is the averaging time scale and $u_i'' = \overline{U}_i - \langle \overline{U}_i \rangle^{\text{AVG}}$ is the fluctuating velocity with respect to the exponentially weighted average. Exponentially weighted average turbulent stresses $\langle \tau_{ij} \rangle^{\text{AVG}}$ for the LES are defined similarly.

To achieve consistency between the two solutions, it is required that the exponentially weighted average quantities and the Reynolds-averaged quantities are approximately equal, e.g., $\langle \overline{U}_i \rangle^{\text{AVG}} \approx \langle U_i \rangle$. The regions which are well-resolved by the LES mesh are classified as LES regions where the LES solution should dominate, and the under-resolved regions are called RANS regions where the RANS solution shall be dominant. Consistency in these selected subdomains is enforced via the drift forces Q_i^L (in the filtered equations) and Q_i^R (in the Reynolds-averaged equations); they defined as follows:

$$Q_i^L = \begin{cases} (\langle U_i \rangle - \langle \overline{U}_i \rangle^{\text{AVG}})/T^{(\text{L})} + G_{ij} u_j''/T^{(\text{G})} & \text{(RANS regions)} \\ 0 & \text{(LES regions)} \end{cases} \tag{3}$$

and

$$Q_i^R = \begin{cases} 0 & \text{(RANS regions)} \\ (\langle \overline{U}_i \rangle^{\text{AVG}} - \langle U_i \rangle)/T^{(\text{R})} & \text{(LES regions)} \end{cases} \tag{4}$$

where

$$G_{ij} = \frac{\tau_{ij} - \langle \tau_{ij} \rangle^{\text{AVG}}}{\langle \tau_{kk} \rangle^{\text{AVG}}}, \tag{5}$$

with the relaxation time scales $T^{(\text{L})}$, $T^{(\text{G})}$, and $T^{(\text{R})}$.

Similarly, to ensure consistency in the well-resolved (LES) regions the turbulent RANS quantities are relaxed towards the corresponding LES quantities via the added drift terms.

In Eq. (3), the term $(\langle U_i \rangle - \langle \overline{U}_i \rangle^{\text{AVG}})/T^{(\text{L})}$ is to make the mean LES velocity consistent with the RANS results in the under-resolved region, while the term $G_{ij} u_j''/T^{(\text{G})}$ enhances or damp the filtered velocities in this region to achieve consistency in terms of turbulent stresses. Although theoretically

*Presenting author

this is an elegant approach, the numerical implementation requires care, mainly since only three velocity components can be directly altered to achieve agreement for the six independent Reynolds stresses.

We have explored several strategies to overcome this difficulty and one of the successful approaches is to enforce the consistency by modifying the mean turbulent stresses in the LES, as opposed to scaling the fluctuations as in the original formulation. This is achieved by correcting the Reynolds stress according to the consistency requirement, and by formulating the relaxation forces accordingly, i.e., by using the following expression for Q_i^L in lieu of Eq. (3):

$$Q_i^L = \begin{cases} -\frac{\partial}{\partial x_j}(\tau_{ij}^{\mathrm{corr}}) & \text{in RANS regions} \\ 0 & \text{in LES regions} \end{cases} \quad (6)$$

with

$$\tau_{ij}^{\mathrm{corr}} = \tau_{ij} - \left(\langle u_i'' u_j'' \rangle^{\mathrm{AVG}} + \langle \tau_{ij}^{\mathrm{sgs}} \rangle^{\mathrm{AVG}} \right), \quad (7)$$

where $\langle u_i'' u_j'' \rangle^{\mathrm{AVG}}$ and $\langle \tau_{ij}^{\mathrm{sgs}} \rangle^{\mathrm{AVG}}$ are resolved and averaged modeled turbulent stresses in LES, respectively.

To provide good predictions in the near wall region, a Reynolds Stress Transport Model with Elliptic Relaxation (RSTM-ER) proposed by Durbin [2] was employed. Due to its physically motivated non-localness, the RSTM-ER is better suited for separating flows than local models or models based on wall functions. Therefore, it is a very suitable candidate for closure of the RANS part in the consistent hybrid LES/RANS framework, and it was implemented for the work presented here.

NUMERICAL SIMULATION

To appraise the performance of the modified formulation, the numerical simulations are conducted for flow over periodic hills, and the results are compared to those obtained with the original formulation presented in ref. [4].

Figure 1 shows comparisons of the mean velocity profiles obtained using direct forcing and using original relaxation forcing with the benchmark results of Breuer et al. [1]. It can be seen that the modified formulation performs better in the region after the separating point.

Figure 2 presents the same comparison with the original formulation for turbulent kinetic energy. Again, improved performance is observed in the circulation region.

CONCLUSION

A consistent hybrid LES/RANS framework previously proposed by Xiao and Jenny [4] has been modified to better exploit the advantages offered by more advanced RANS models with better Reynolds stress prediction, such as the Reynolds stress transport model of Durbin [2]. In the new formulation, the LES turbulent stress is directly modified in the under-resolved regions. The modified formulation represents a feasible and robust way to overcome the difficulty of enforcing componentwise Reynolds stress consistency with the original relaxation strategy. Preliminary evaluations for flow over periodic hills have shown promising results.

REFERENCES

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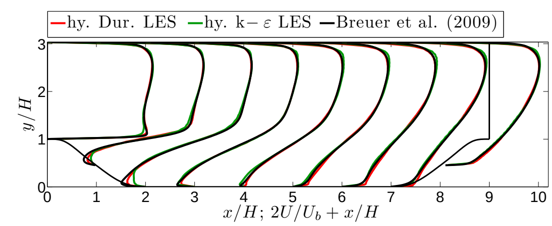

Figure 1: Mean velocity profiles in flow over periodic hills at nine streamwise locations. The profiles obtained by using the modified formulation are compared with those from the original formulation and with the benchmark results of Breuer et al. [1].

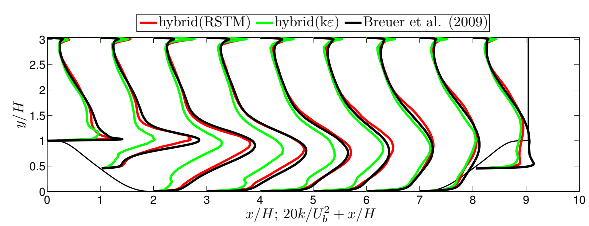

Figure 2: Turbulent kinetic energy profiles in flow over periodic hills. The profiles obtained by using the modified formulation are compared with those from the original formulation and with the benchmark results of Breuer et al. [1].