AIAA2017-0081

Hyperbolic Navier-Stokes Method for High-Reynolds-Number Boundary-Layer Flows

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Cradle

A Greatest Tip for Healthier Life

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“If you suffer from high blood pressure/cholesterol/sugar, lose weight, and you’ll be healthier.”

[1] Blackburn G. (1995). Effect of degree of weight loss on health benefits. Obesity Research 3: 211S-216S

[2] “Clinical Guidelines on the Identification, Evaluation, and Treatment of Overweight and Obesity in Adults”: http://www.nhlbi.nih.gov/guidelines/obesity/ob_gdlns.pdf

Great, but doesn’t tell us how to reduce weight…..

“How to lose weight?”

Approaches are not always non-intuitive:
eat breakfast, drink water, turn off TV, etc.

A Greatest Tip in Healthier CFD

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"If your CFD code doesn't work, add more artificial viscosity, and it'll work."

Increase Q!

$$\Phi(\mathbf{u}_L, \mathbf{u}_R) = \frac{1}{2}(\mathbf{f}_L + \mathbf{f}_R) - \frac{1}{2}\underline{\mathbf{Q}}(\mathbf{u}_R - \mathbf{u}_L)$$

Artificial viscosity / Dissipation / High-frequency damping

"How, and how much to add?"

Many different approaches:

Artificial viscosity, Riemann solvers, artificial upwinding (FVS), large edge-term contribution, compact viscous stencil, etc.

Hyperbolic Navier-Stokes

Since JCP2007

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HNS20 AIAA 2016-1101

$$\begin{aligned}
\mathrm{div}(\rho \mathbf{v}) &= \mathrm{div}\,\mathbf{r}, \\
\mathrm{div}(\rho \mathbf{v} \otimes \mathbf{v}) + \mathrm{grad} p - \mathrm{div} \boldsymbol{\tau} &= 0, \\
\mathrm{div}(\rho \mathbf{v} H) - \mathrm{div}(\boldsymbol{\tau} \mathbf{v}) + \mathrm{div} \mathbf{q} &= 0, \\
-\,\mathrm{grad}\,\mathbf{v} + \mathbf{g}/\mu_v &= 0, \\
\mathrm{grad}\left(\frac{T}{\gamma(\gamma-1)}\right) + \mathbf{q}/\mu_h &= 0, \\
-\,\mathrm{grad} \rho + \mathbf{r}/\nu_\rho &= 0.
\end{aligned}$$

- Simplified viscous discretization/solver
- Improved convergence
- Accurate gradients on irregular grids
- Higher-order inviscid approximation

Gradient variables

$$\nabla \rho_j = \frac{\mathbf{r}_j}{\nu_\rho},$$

$$\nabla \mathbf{v}_j = \frac{\mathbf{g}_j}{(\mu_v)_j},$$

$$\nabla T_j = -\gamma(\gamma-1)\frac{\mathbf{q}_j}{(\mu_h)_j}.$$

Refs. (3D HNS):

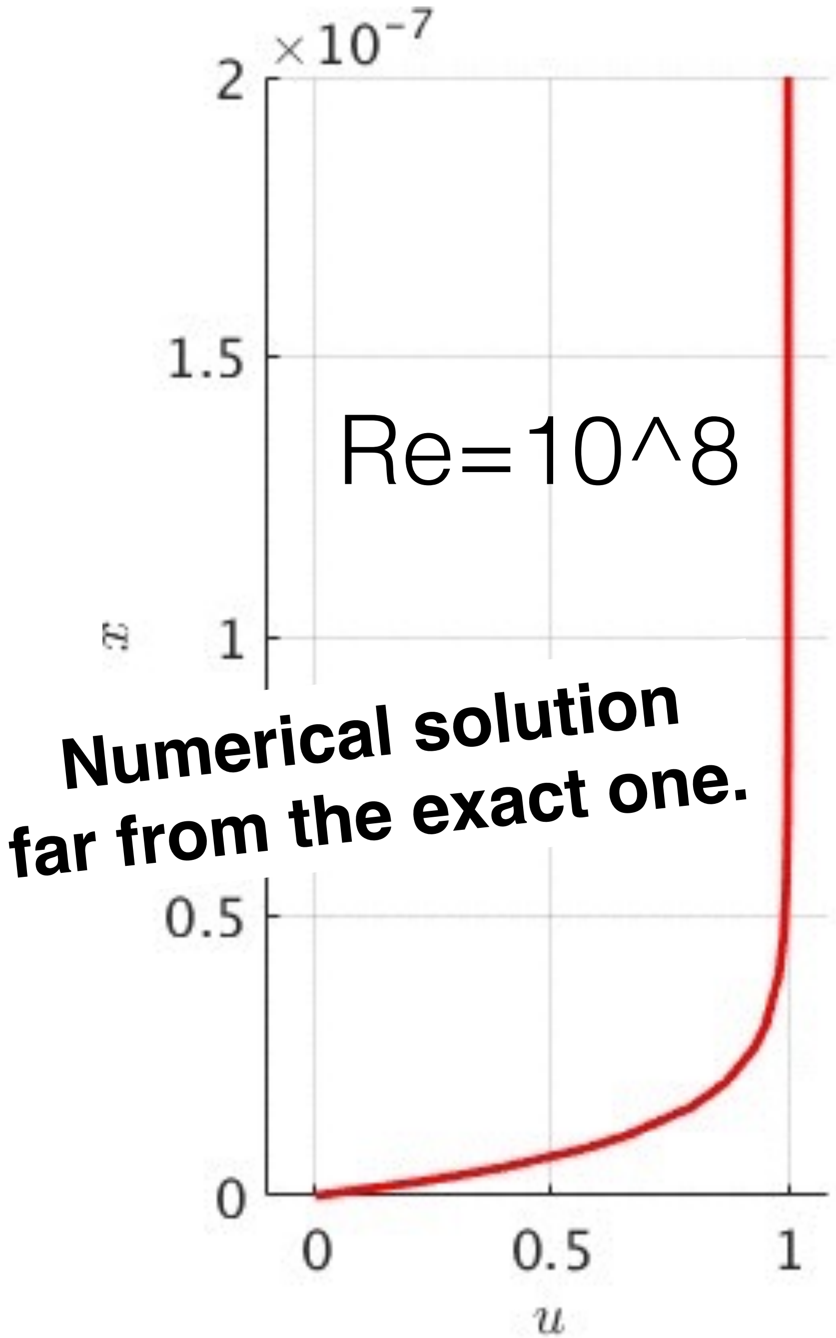
AIAA 2016-3969

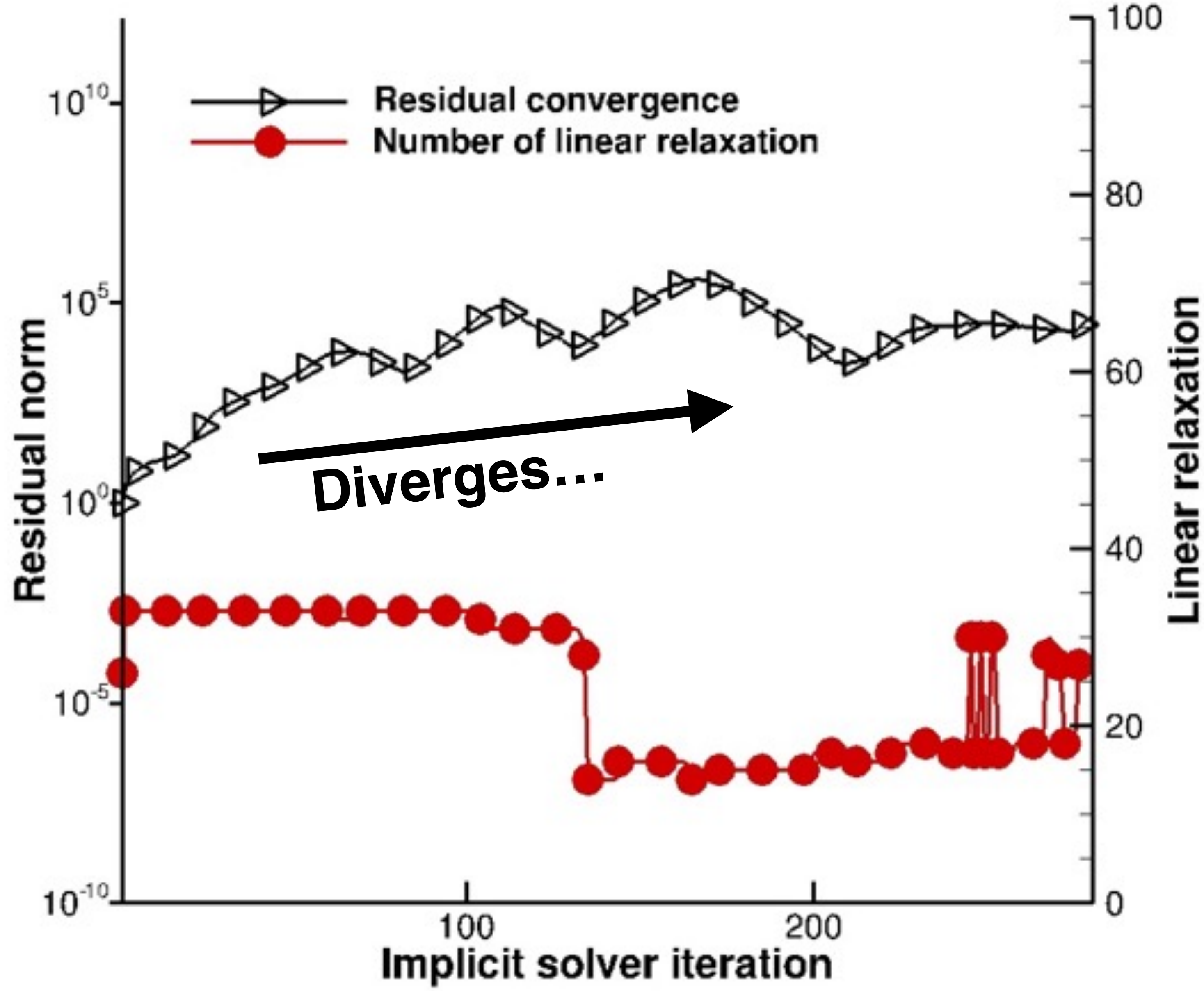
AIAA 2016-1101

Website:

hiroakinishikawa.com/fohsm/

Inaccuracy and Divergence





Serious problem for practical viscous-flow application…

No problem in inviscid limits; this is a problem for BL.

Just Increase Dissipation!

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"If your CFD code doesn't work,
add more artificial viscosity, and it'll work."

Case of Conventional Scheme

Central scheme: $a\dfrac{u_{j+1}-u_{j-1}}{h} - \nu\dfrac{u_{j+1}-2u_j+u_{j-1}}{h^2} = 0$

has the well-known mesh-Re restriction: $Re_h < 2$

Central scheme can be written in the conservative form:

[See cfdnotes.com "Diffusion Scheme for Discontinuous Data" or CF2011]

$$f_{j+1/2} - f_{j-1/2} = 0$$

where

$$\boxed{f_{j+1/2} = \frac{1}{2}(f_{j+1} + f_j) - \frac{a}{Re_h}(u_R - u_L)}$$

$$f_j = a u_j - \nu\frac{u_{j+1}-u_{j-1}}{2h} \qquad u_L = u_j + \frac{h}{2}\frac{u_{j+1}-u_{j-1}}{2h}$$

Indicates that central scheme fails by lack of dissipation for Reh > 2.

Typical remedy is to use dissipative advection scheme, e.g., upwind.

Hyperbolic Scheme

Hyperbolic advection-diffusion system:

$$
\begin{aligned}
\partial_\tau u + \partial_x (au - \nu p) &= 0 \\
(T_r/\nu)\partial_\tau p - \partial_x u &= -p/\nu
\end{aligned}
$$

Relaxation time:

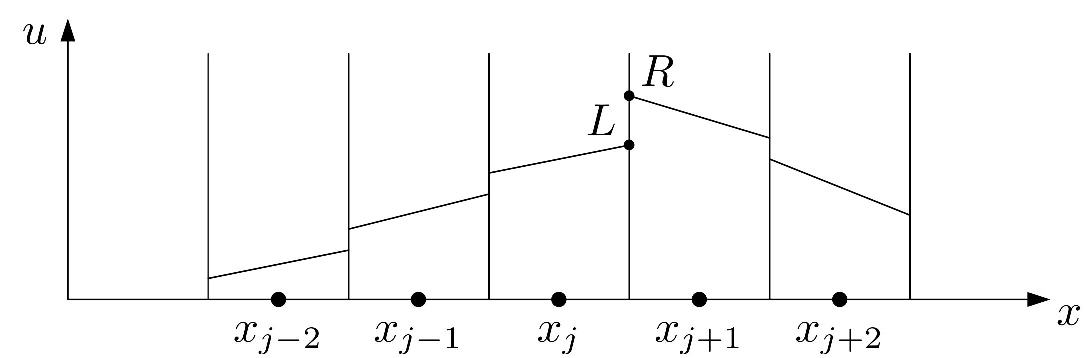
$$T_r = L_r^2/\nu$$

Length scale (JCP2007,2013):

$$L_r = L_d \equiv \frac{1}{2\pi}$$

Node-centered FV with upwind flux:

$$\Phi(\mathbf{u}_L, \mathbf{u}_R) = \frac{1}{2}(\mathbf{f}_L + \mathbf{f}_R) - \frac{1}{2}\mathbf{Q}(\mathbf{u}_R - \mathbf{u}_L)$$



Dissipation matrix |dF/dU| can be constructed (JCP2010), but it is too complicated for the Navier-Stokes…

Dissipation Matrix

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Simplified approach (AIAA2011-3043):

Advective Jacobian + Diffusive Jacobian:

$$\mathbf{Q} = |\mathbf{A}^a| + |\mathbf{A}^d| = \begin{bmatrix} |a| & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a_v & 0 \\ 0 & a_v \end{bmatrix}$$

a_v : Diffusion wave speed

The approach has been successfully extended to 3D NS (AIAA 2016-1101), but the convergence problem was encountered for High-Re BL.

Note: Extra dissipation from advection scheme only helps the first equation.

Nakashima (Cradle) and Baty (U of Strasbourg) pointed out that Lr needs to be reduced for high-Re boundary layers. **Good approach.**

Questions: ***How much to reduce? What does it mean?***

Exploring various approaches,
we found one that works for us…

Forget the dissipation, and look at the error.

1D Boundary Layer Problem

Balanced High-Re Limit

$$a\partial_x u = \nu \partial_{xx} u$$

Exact solution:

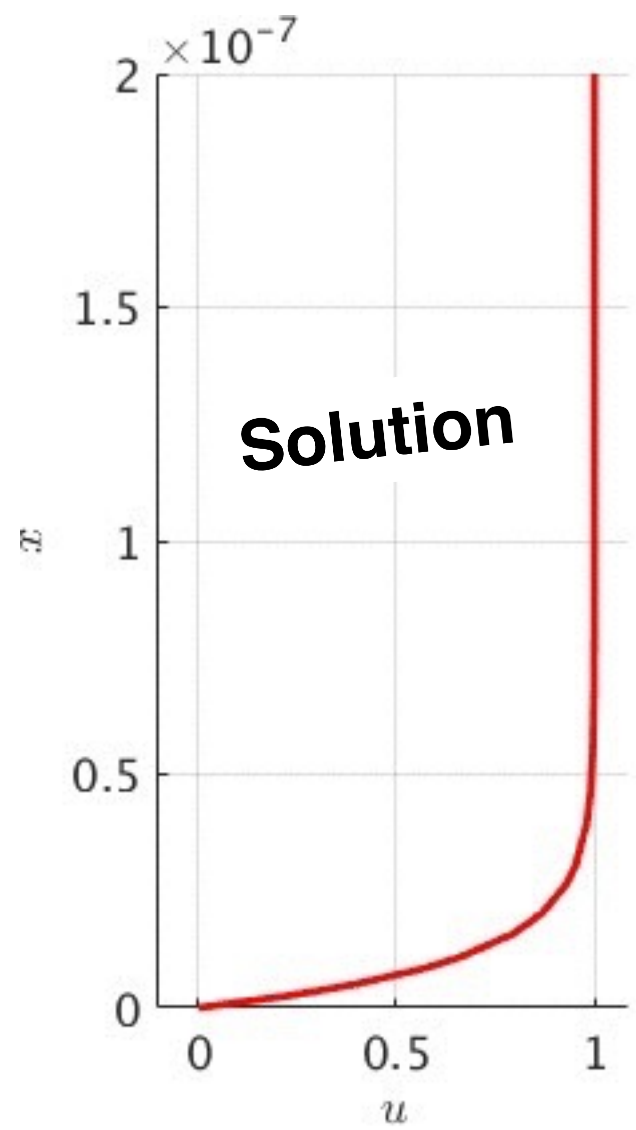
$$u(x) = \frac{1-\exp(Re_x)}{1-\exp(Re)}$$

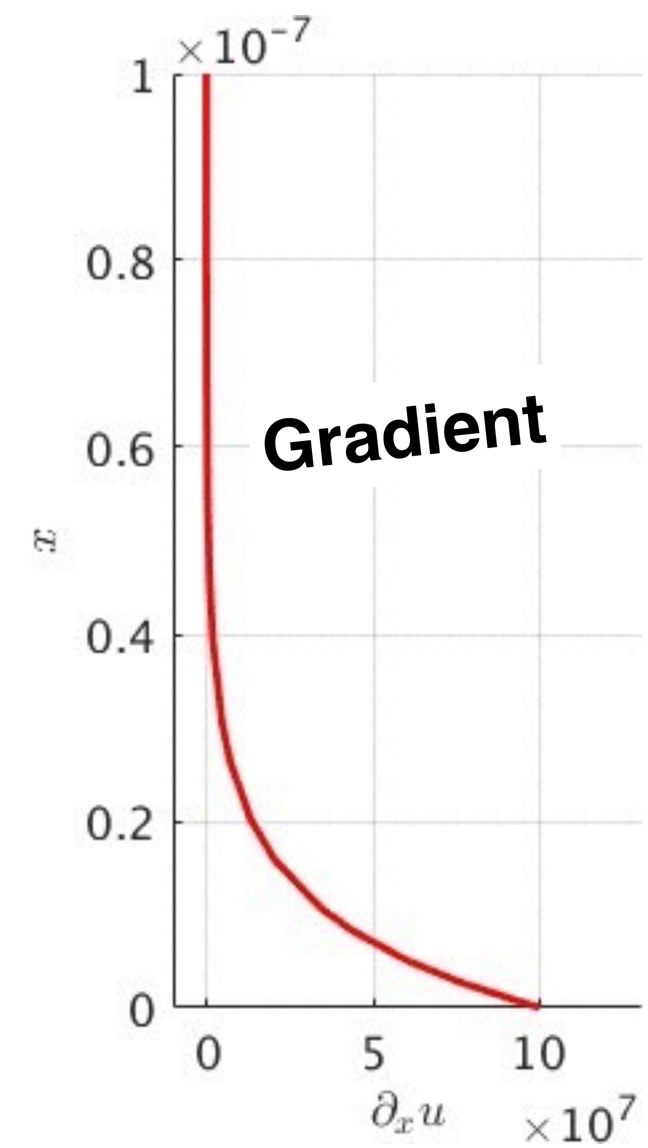
$$= K_1\underline{(1)} + \underline{K_2 \exp(Re_x)}$$

two modes

Reynolds number based on x:

$$Re_x = \frac{ax}{\nu}$$





Modal Analysis for 1st-Order Scheme

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Residual$(\mathbf{u}_j = \mathbf{U}c^j)$ $\longrightarrow$ $\mathbf{MU} = 0$ $\longrightarrow$ det(**M**) = 0

$$
\begin{aligned}
c_1 &= 1 \\
c_2 &= \frac{2Re_{Lr}^2 + 2Re_{Lr}Re_h + 2Re_{Lr} + Re_h + \sqrt{(Re_h + 2Re_{Lr})\left(4Re_{Lr}^2 Re_h + 4Re_{Lr}Re_h + 2Re_{Lr} + Re_h\right)}}{2(Re_{Lr}^2 + 2Re_{Lr} + Re_h)} \\
c_3 &= \frac{2Re_{Lr}^2 + 2Re_{Lr}Re_h + 2Re_{Lr} + Re_h - \sqrt{(Re_h + 2Re_{Lr})\left(4Re_{Lr}^2 Re_h + 4Re_{Lr}Re_h + 2Re_{Lr} + Re_h\right)}}{2(Re_{Lr}^2 + 2Re_{Lr} + Re_h)}
\end{aligned}
$$

where

$$
Re = \frac{a}{\nu}, \quad Re_{L_r} = \frac{aL_r}{\nu}, \quad Re_{L_d} = \frac{aL_d}{\nu}, \quad Re_h = \frac{ah}{\nu}
$$

Accurate modes are c1=1 and c2:

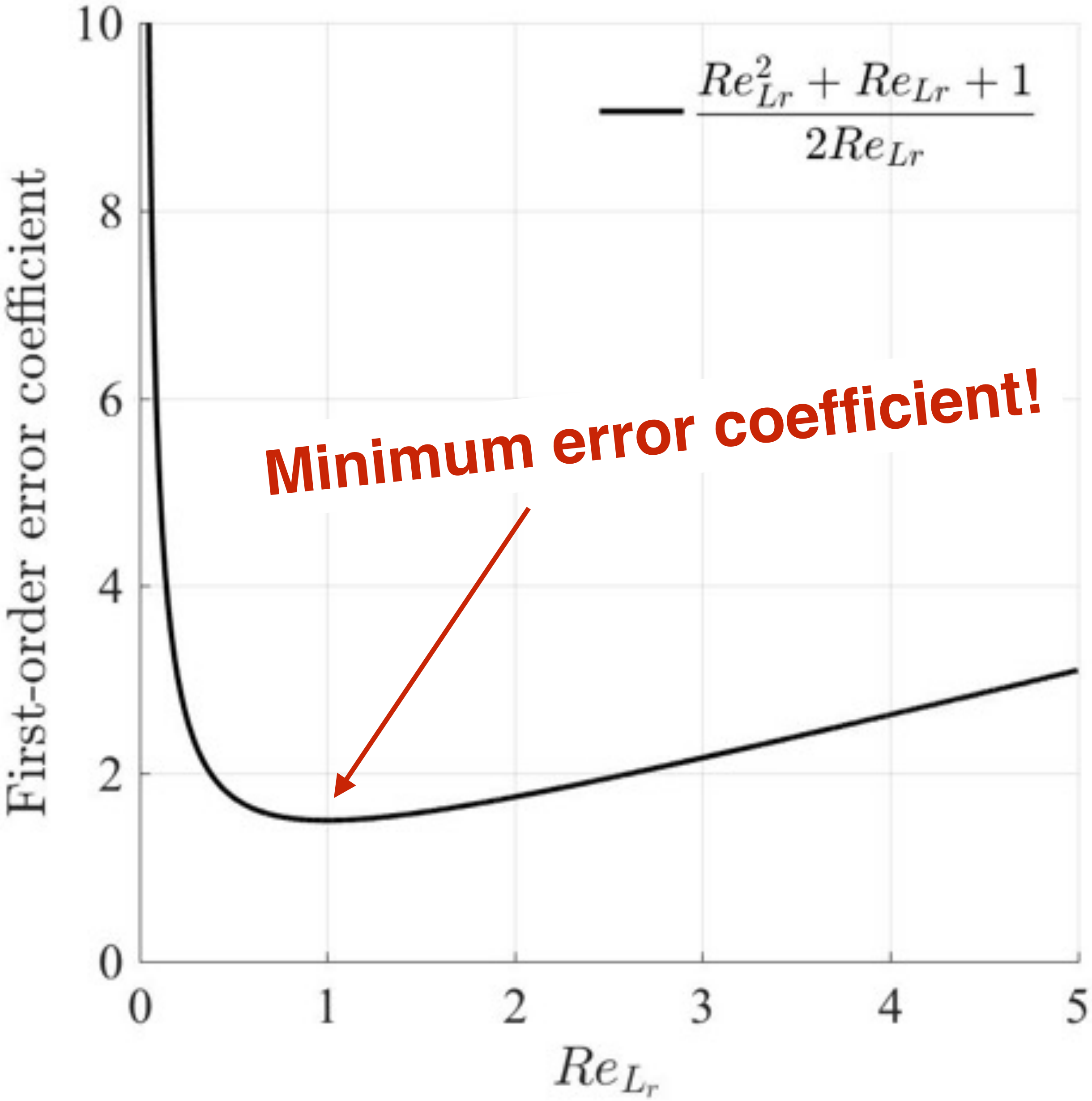
$$
c_2^j = c_2^{\frac{x}{h}} = \underline{\exp(Re_x)} \left(1 - \underline{\boxed{\frac{Re_{Lr}^2 + Re_{Lr} + 1}{2Re_{Lr}}} Re_x Re_h} + O(h^2)\right)
$$

1st-order error

"Oh, no… This is a problem…"

Minimize the error

For $Re = \frac{a}{\nu} \to \infty \longrightarrow Re_{L_r} = L_r Re \to \infty$ if $L_r = L_d \equiv \frac{1}{2\pi}$



Minimum error at

$$\frac{\partial}{\partial Re_{Lr}} \left(\frac{Re_{Lr}^2 + Re_{Lr} + 1}{2Re_{Lr}} \right) = 0$$

The solution is

$$Re_{Lr} = 1$$

which means

$$\boxed{L_r = \frac{1}{Re}}$$

Dissipation retained

$$L_r = L_d \equiv \frac{1}{2\pi} \longrightarrow a_v = \frac{\nu}{L_r} = 2\pi\nu = \frac{2\pi a}{Re} \to 0$$

$$\mathbf{Q} = |\mathbf{A}^a| + |\mathbf{A}^d| = \begin{bmatrix} |a| & 0 \\ 0 & 0 \end{bmatrix} + \cancelto{0}{\begin{bmatrix} a_v & 0 \\ 0 & a_v \end{bmatrix}}$$

$$L_r = \frac{1}{Re} \longrightarrow a_v = \frac{\nu}{L_r} = \nu Re = \nu\frac{a}{\nu} = a$$

$$\mathbf{Q} = |\mathbf{A}^a| + |\mathbf{A}^d| = \begin{bmatrix} |a| & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a_v & 0 \\ 0 & a_v \end{bmatrix}$$

Dissipation from hyperbolic diffusion stays finite for large Re, and acts as additional dissipation to fight against high-frequency errors.

Iterative Convergence (2nd-order)

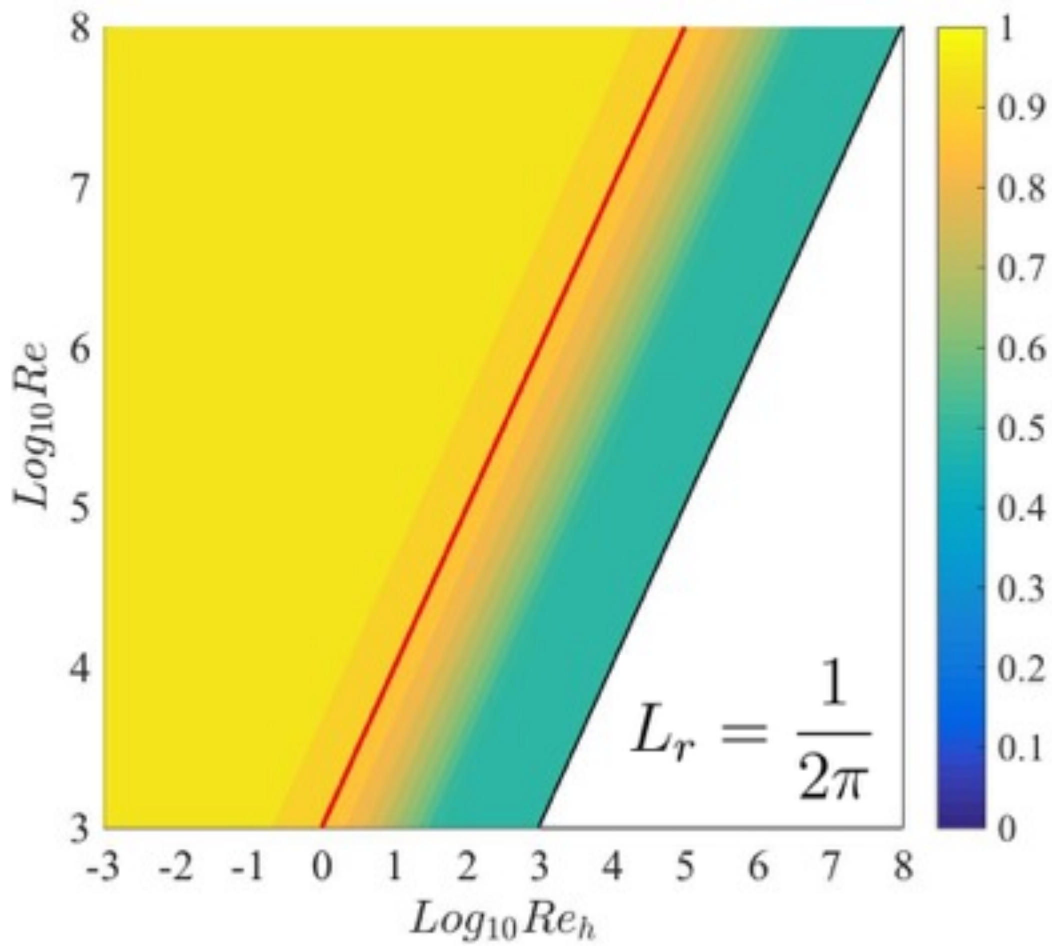
Implicit solver

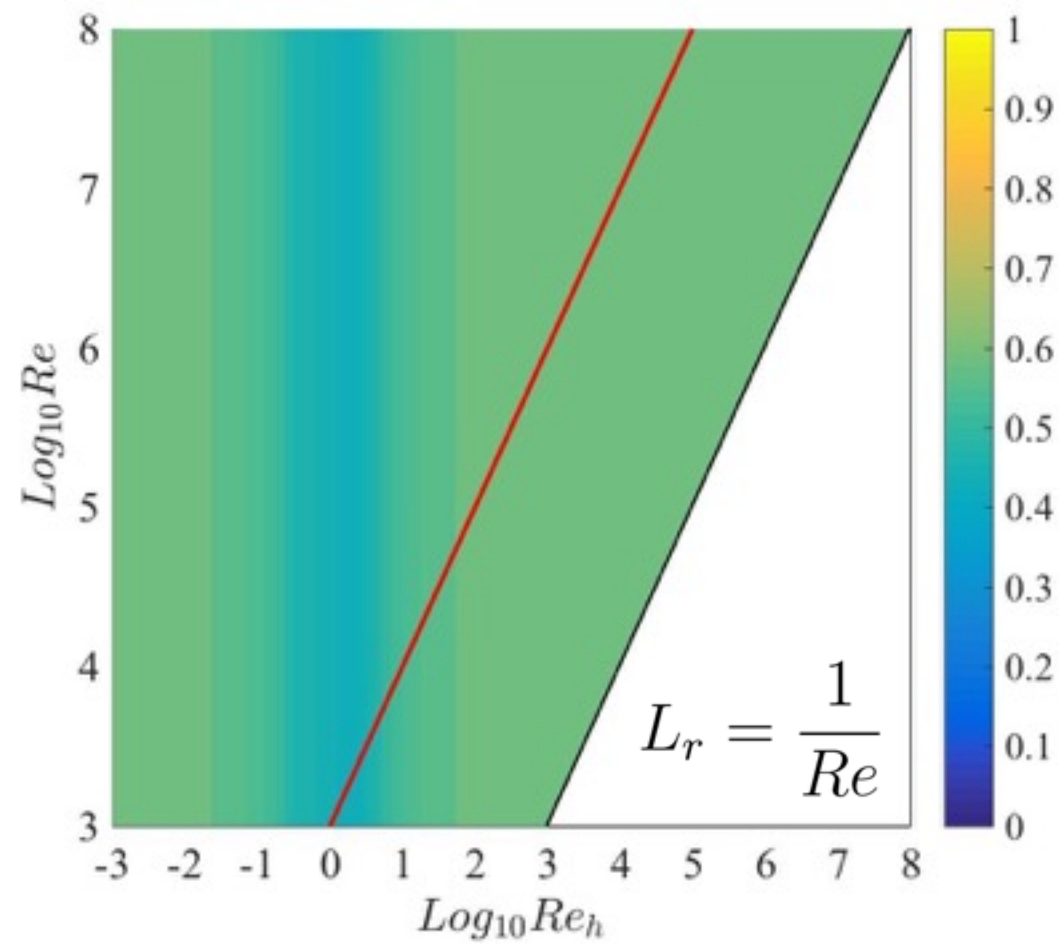
$$\mathbf{U}^{k+1} = \mathbf{U}^k + \Delta\mathbf{U}$$
$$\mathbf{J}\Delta\mathbf{U} = -\mathbf{Res}(\mathbf{U}^k)$$

$\longrightarrow$

Fourier transform

$$\mathbf{U}_0^{k+1} = \left(\mathbf{I} - \mathbf{J}^{-1}\mathbf{R}\right)\mathbf{U}_0^k$$





Extra dissipation also resolves convergence issue.

Greatest tip works!

"If your CFD code doesn't work,
add more artificial viscosity, and it'll work."

"Add more dissipation by Lr=1/Re! "

$$\begin{bmatrix} |a| & 0 \\ 0 & 0 \end{bmatrix} + \underline{\begin{bmatrix} a_v & 0 \\ 0 & a_v \end{bmatrix}}$$

Second-Order Schemes

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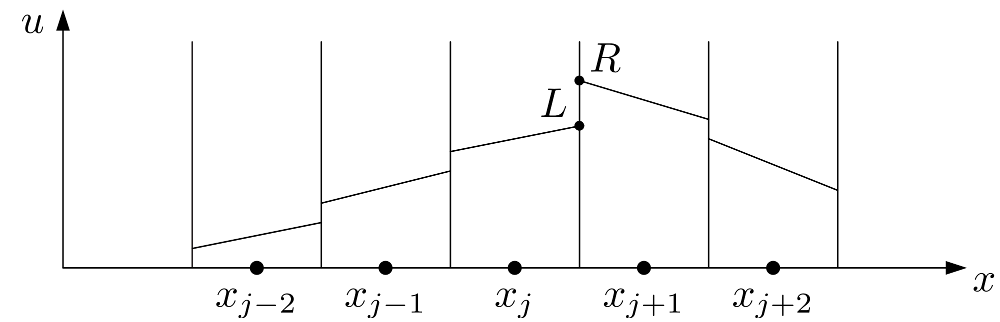
Linear reconstruction to the face:

$$\mathbf{u}_L = \mathbf{u}_j + \frac{\Delta x_j}{2}(\partial_x \mathbf{u})_j, \quad \mathbf{u}_R = \mathbf{u}_{j+1} - \frac{\Delta x_{j+1}}{2}(\partial_x \mathbf{u})_{j+1},$$



Scheme-I: Linear LSQ gradient for all.

$$(\partial_x \mathbf{u})_j = \frac{\displaystyle\sum_{k \in \{k_j\}} (\mathbf{u}_k - \mathbf{u}_j)(x_k - x_j)}{\displaystyle\sum_{k \in \{k_j\}} (x_k - x_j)^2}$$

Scheme-IQ: Compact quadratic LSQ gradient for u.

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$$(\partial_x u)_j = \frac{\displaystyle\sum_{k \in \{k_j\}} \left(u_k - u_j - \frac{1}{2\nu}(\partial_x p)_j (x_k - x_j)^2 \right)(x_k - x_j)}{\displaystyle\sum_{k \in \{k_j\}} (x_k - x_j)^2}$$

Scheme-II:

$$(\partial_x u)_j = p_j / \nu$$

Issues remain.
See paper.

Scheme-IQ achieves 3rd-order accuracy in advection limit.

Second-Order Correction

Modal analysis gives:

$$(c-1)\left[K_3c^3 + K_2c^2 - K_1c + K_0\right] = 0,$$

where

$$K_3 = Re_{Lr}\left(Re_{Lr}+3\right)c^3, \quad K_2 = Re_{Lr}^2 + (7-4Re_h)Re_{Lr} + 4Re_h,$$
$$K_1 = 5Re_{Lr}^2 + (12Re_h+7)Re_{Lr} + 4Re_h, \quad K_0 = 3Re_{Lr}\left(Re_{Lr}-1\right).$$

Minimize the number of negative roots:

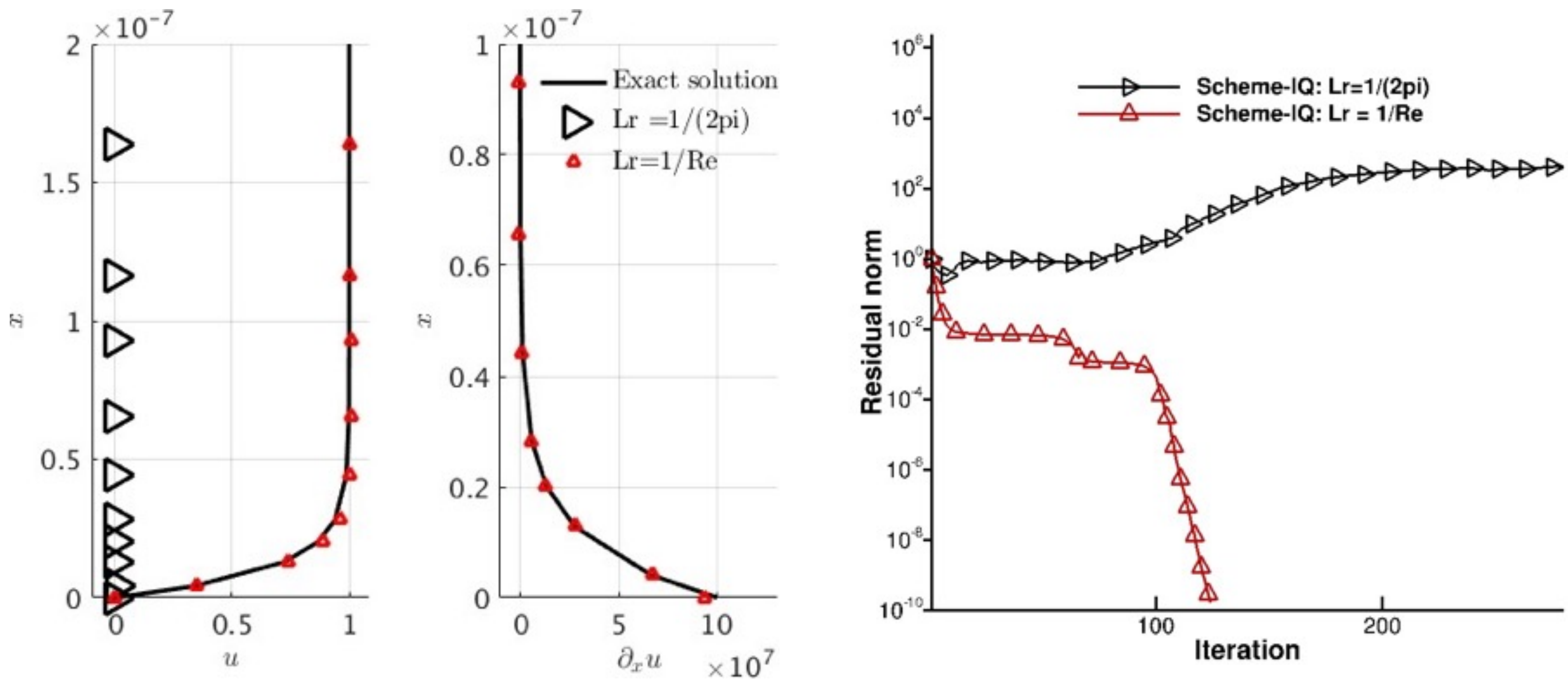
$$L_r = \frac{K}{Re}, \quad K = \max\left(1, 4Re_h - \frac{7}{2}\right)$$

K is the 2nd-order correction to Lr = 1/Re, which has been found effective for under-resolved grids.

Results in One Dimension

Re = 100 million.

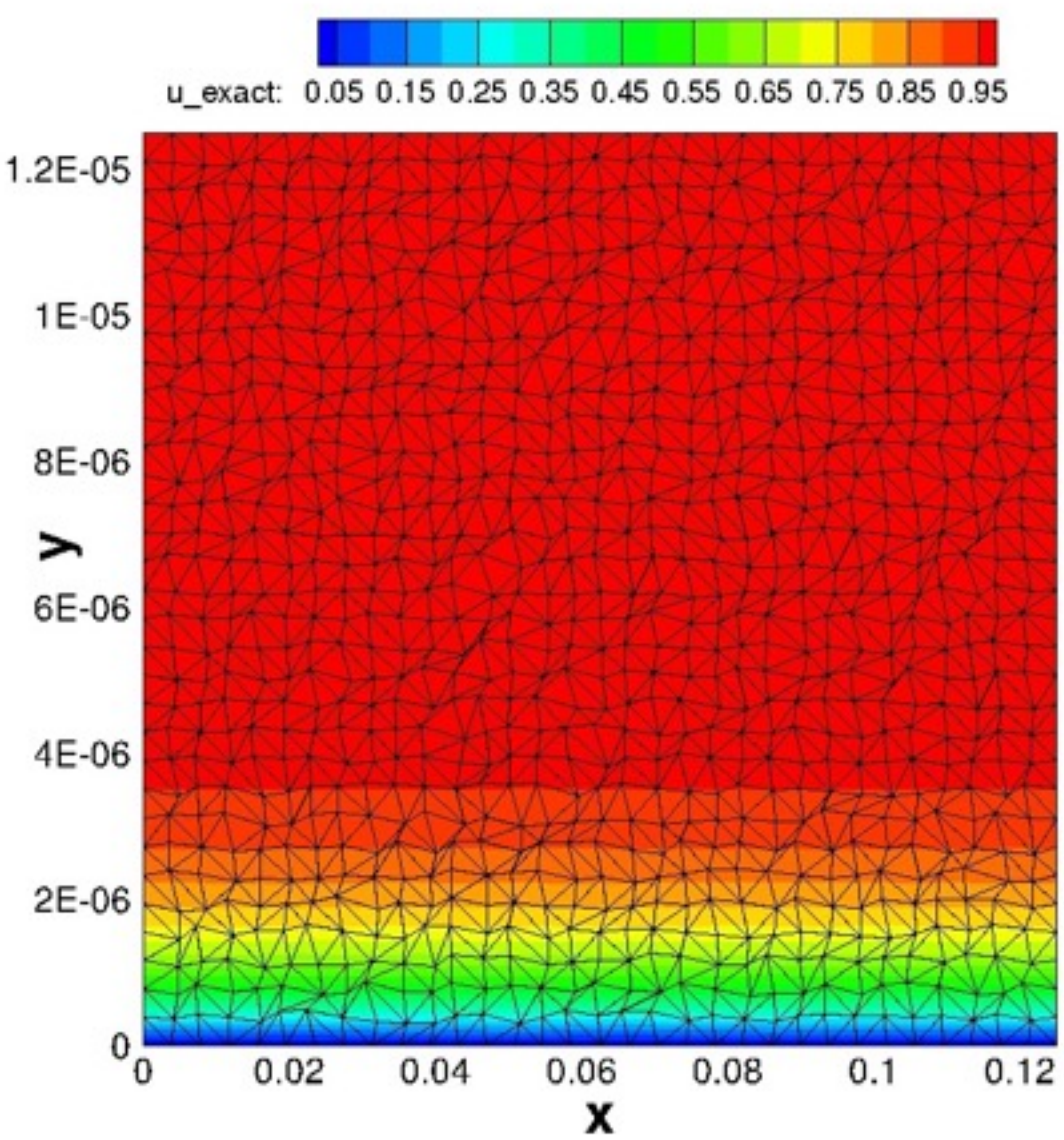
Scheme-IQ converges rapidly and produce accurate solutions.

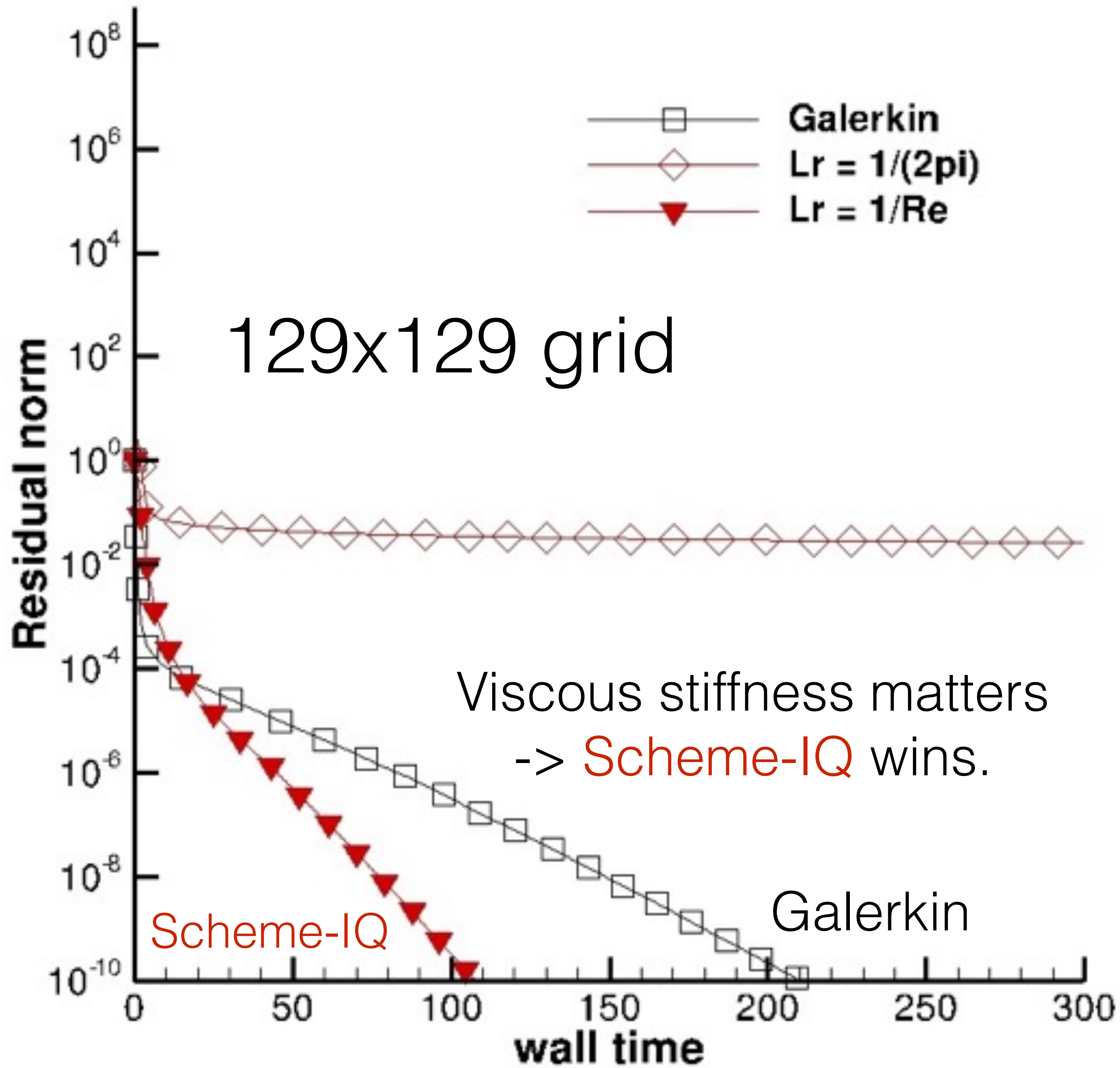


Results for strong/weak BCs and for viscous Burgers in the paper.

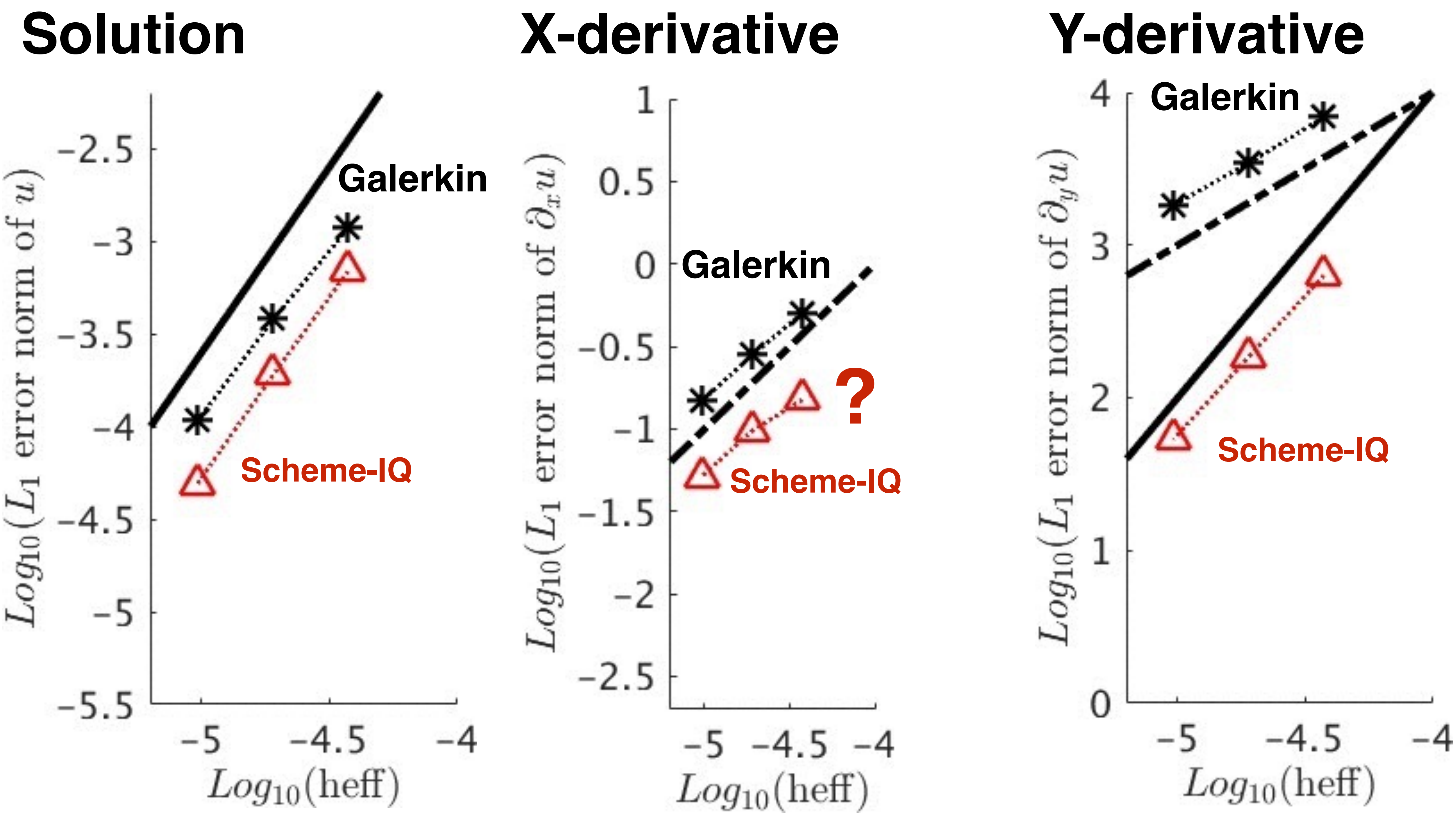
Results in Two Dimension

1D BL in a 2D doman. Re = 1 million.
Scheme-IQ converges rapidly and produce accurate solutions.





Results in Two Dimension



Comparison: Galerkin and Hyperbolic with Lr=1/Re.

Results in Two Dimension

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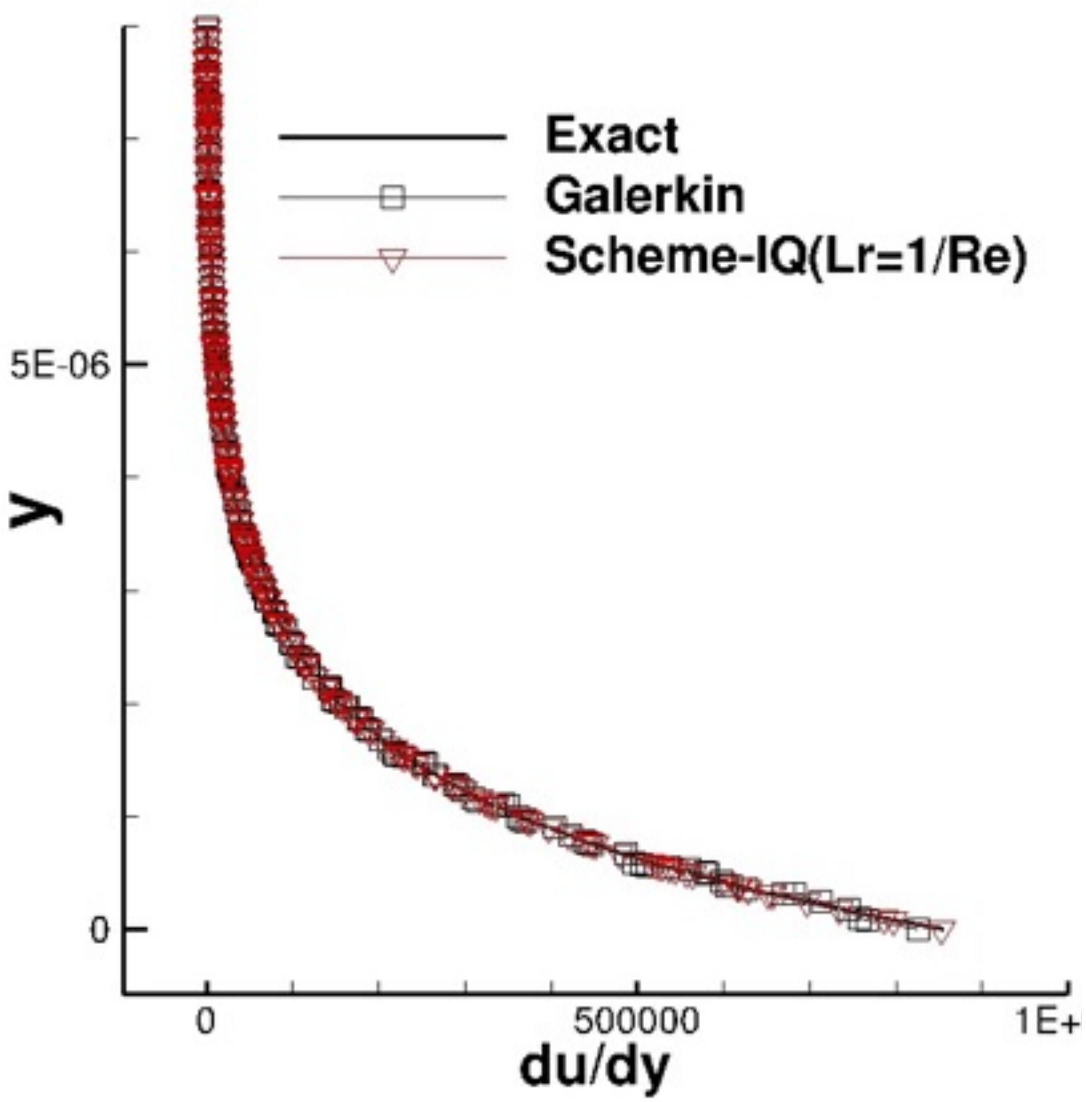
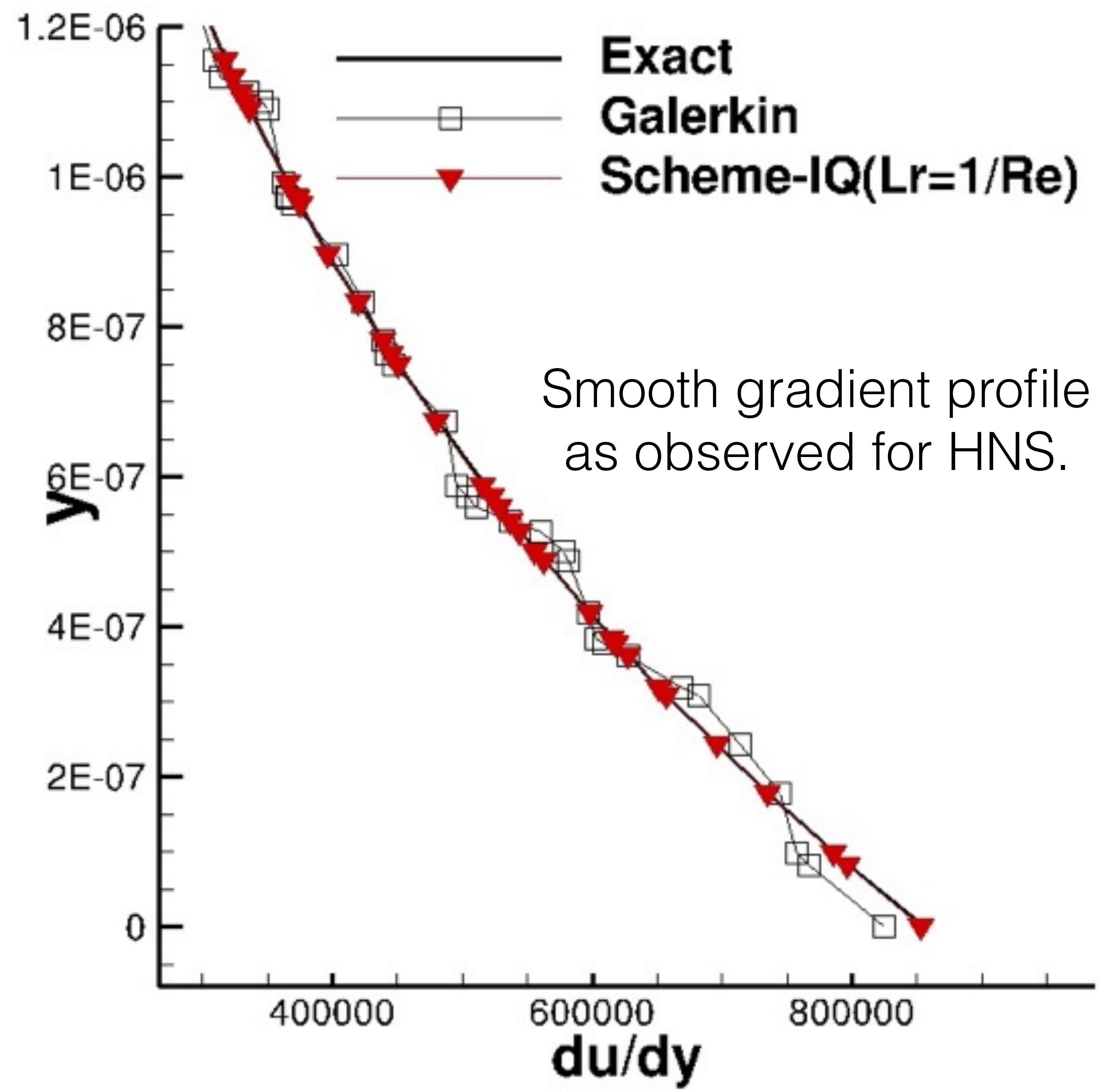
Comparison of Gradient (viscous stress)





Smooth gradient profile as observed for HNS.

Extensions to HNS

Three lengths for HNS20: $T_\rho = \dfrac{L_\rho^2}{\nu_\rho}, \quad T_v = \dfrac{L_v^2}{\nu_v}, \quad T_h = \dfrac{L_h^2}{\nu_h}$

Set L = 1/Re for all in high-Re cases:

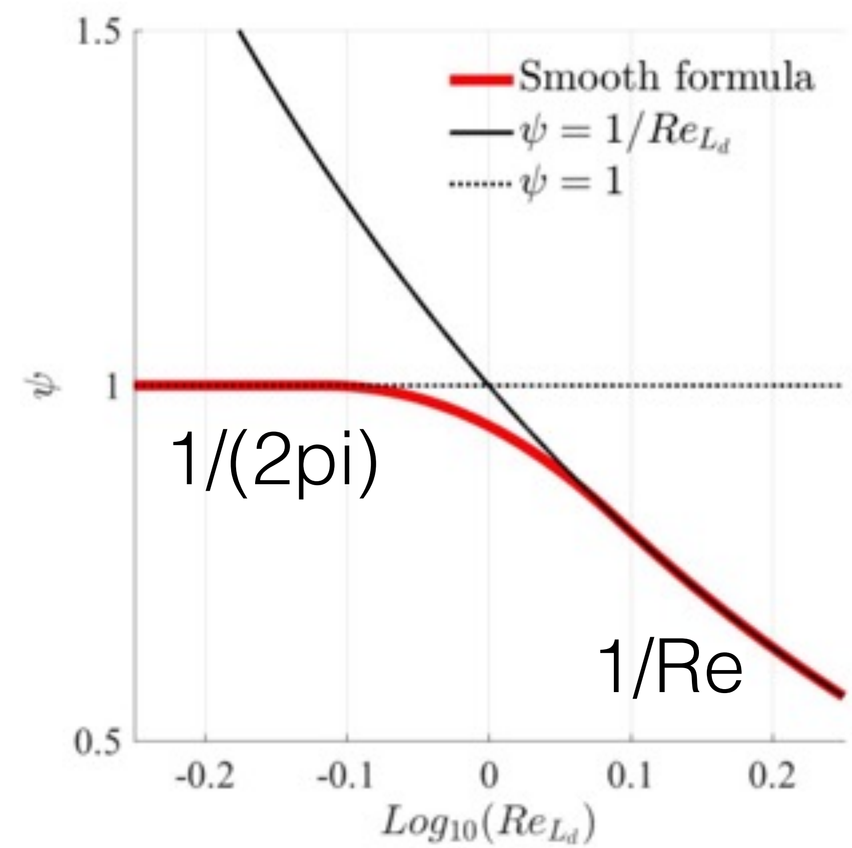
$$L_v = \psi(Re^v_{L_d})L_d, \qquad Re^v_{L_d} = \frac{|u|L_d}{\nu_v} \qquad L_d \equiv \frac{1}{2\pi}$$

Switching function:

$$\psi(Re^v_{L_d}) = \min\left(1, \frac{K}{Re^v_{L_d}}\right)$$

K: 2nd-order correction for under-resolved grids.

L = 1/(2pi) for small R
L = 1/Re for large Re.



Options

1. Local Reynolds number

$$\psi(Re^{v}_{L_d}) = \min\left(1, \underline{\frac{K}{Re^{v}_{L_d}}}\right)$$

2. Free-stream Reynolds number

$$\psi_{\infty}(Re^{\nu_{\infty}}_{L_d}) = \min\left(1, \underline{\frac{K}{Re^{\nu_{\infty}}_{L_d}}}\right)$$

3. Approximate function

$$\psi(Re^{v_{\infty}}_{L_d}) = \frac{1}{\sqrt{Re^{v_{\infty}}_{L_d}}}$$

Results for 3D HNS (FUN3D)

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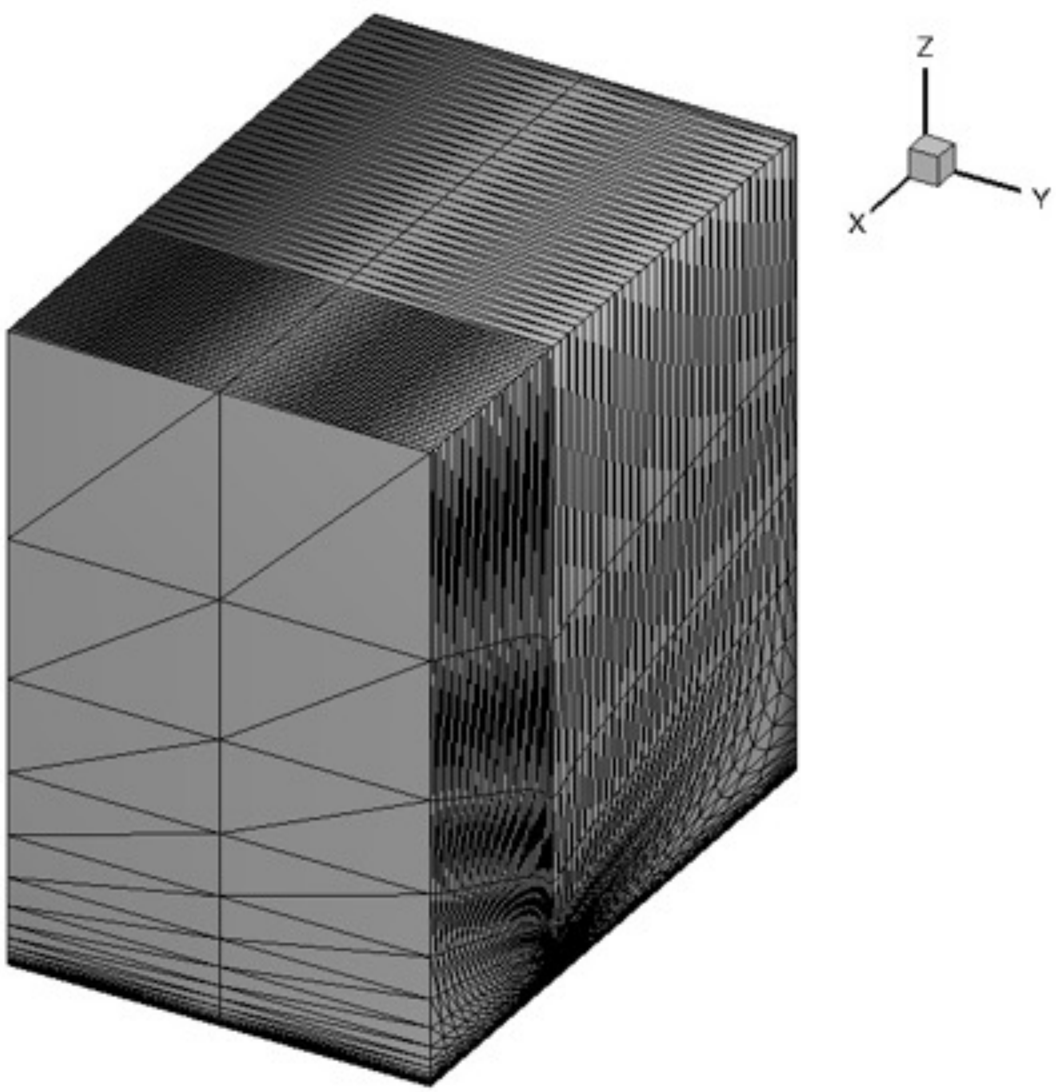
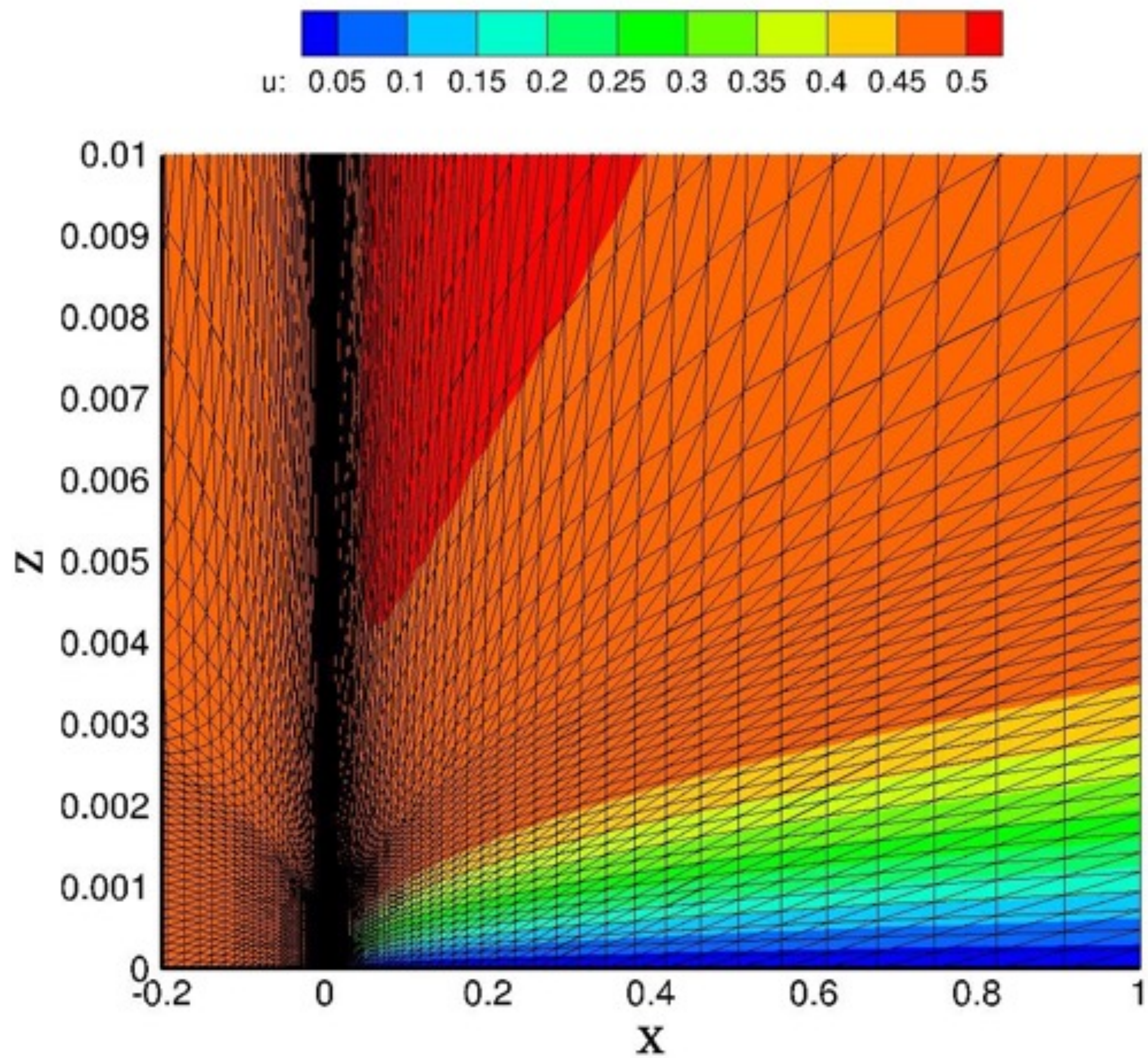
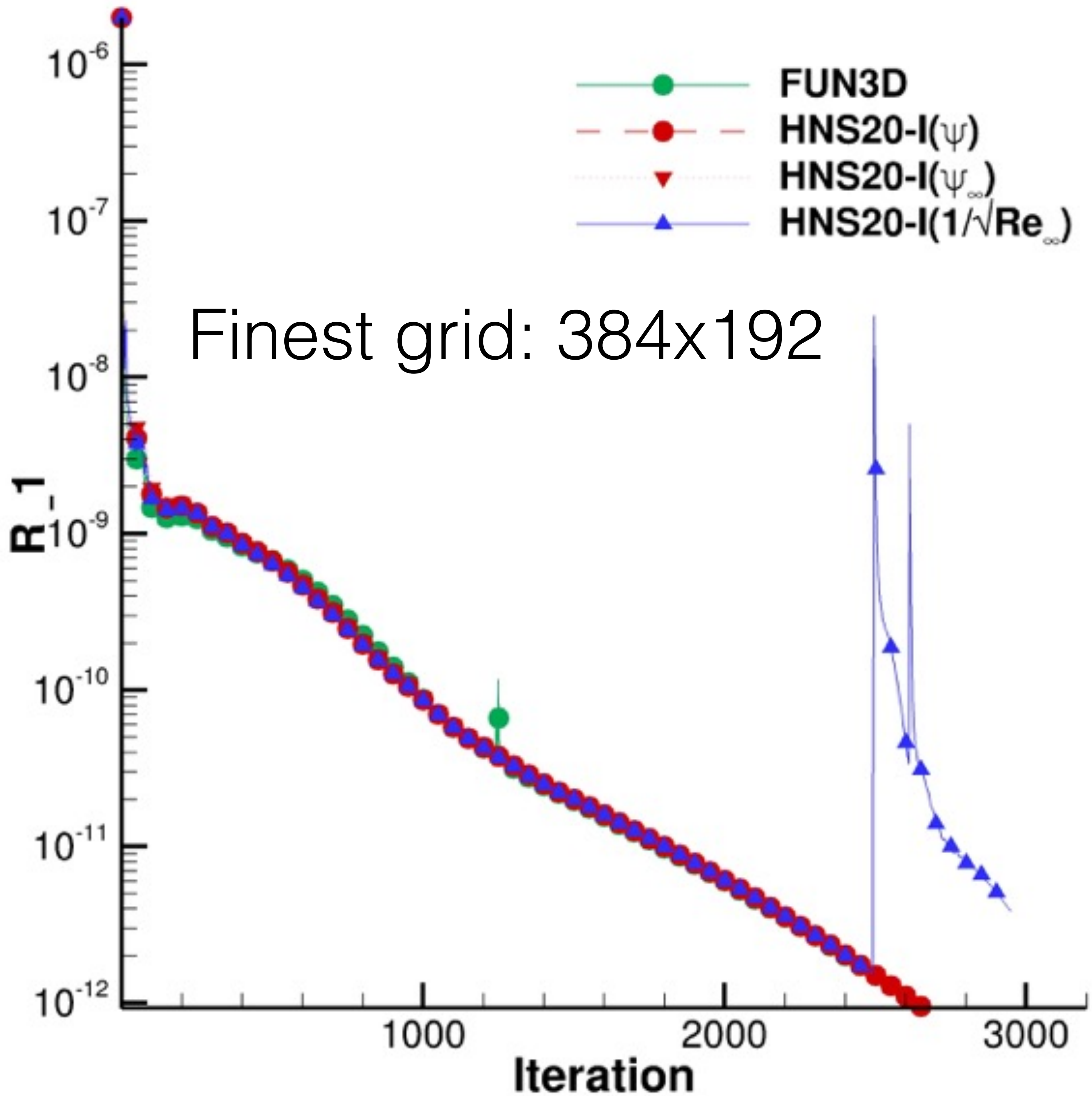
Laminar flat plate at Re = 1 million





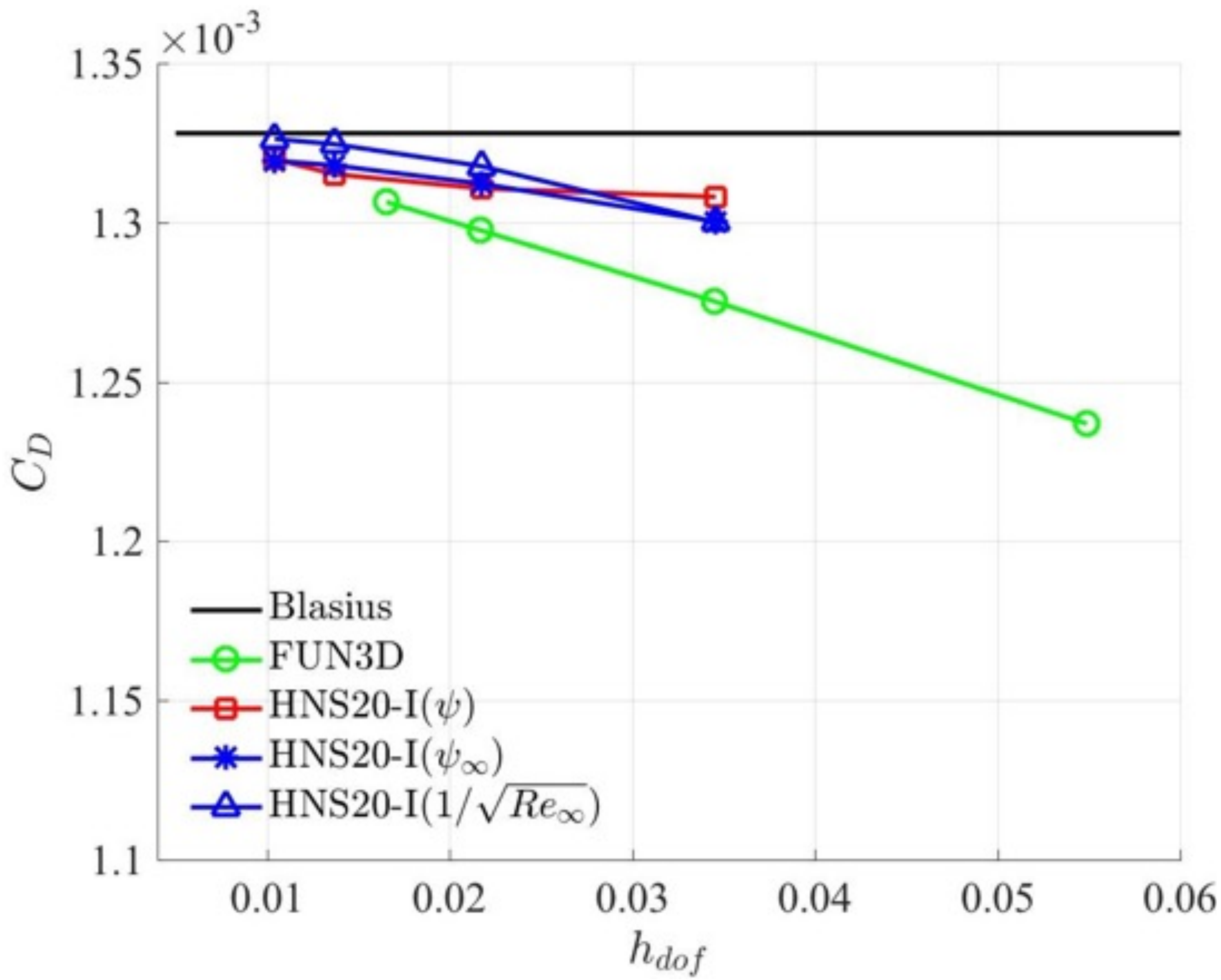
Results for HNS



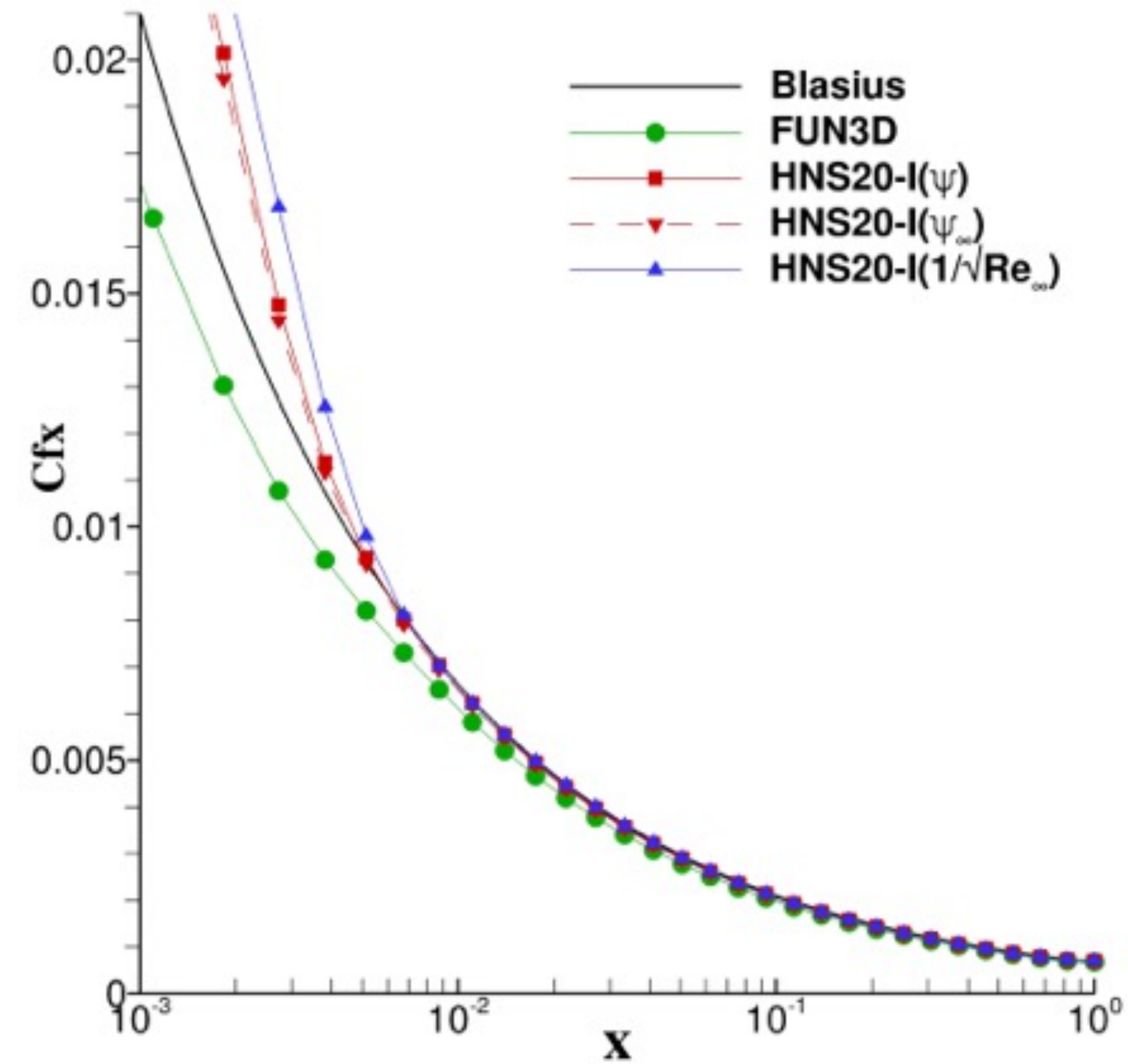
- 4 grids
  64x32, 128x64
  256x128, 384x192

- 15 linear relaxations.
- CFL = 200
- HNS20 Jacobian with AD.

Results for HNS



64x32



Superior not only in Cd, but also in the local skin friction.

Conclusions

- **Lr = 1/Re for high-Re boundary-layer flows.**

Improves accuracy and iterative convergence.
Dissipation recovered, which vanishes otherwise.

- **Lr = 1/Re may be evaluated at free-stream for NS.**

Simple to implement.

HNS can be made useful for accurate and efficient boundary-layer calculations.

Current and Future work

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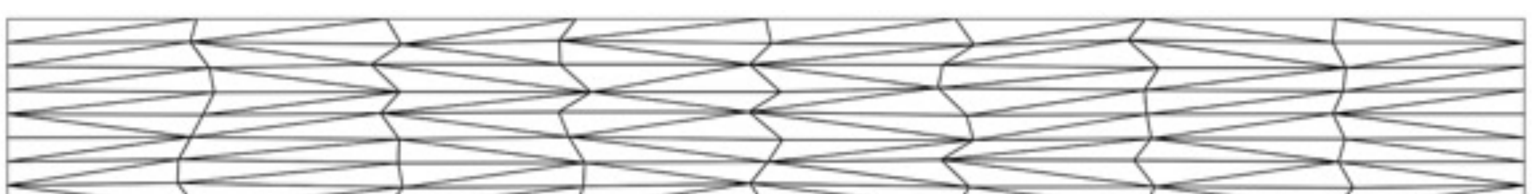
- Extend HNS20 to high-Re turbulent flows.

NASA TMR test cases: Hemisphere cylinder, OM6.

- Issues remain for Scheme-II. Details in the paper. *AIAA2017-0081*

Problems with irregular stretched grids and 3D viscous cases.

- Accuracy deterioration in gradient: one-order lower.



gradient in this direction.

A Greatest Tip in CFD

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"If your CFD code doesn't work,
add more artificial viscosity, and it'll work."

Keep adding dissipation until it works!

NOTE: I'm just kidding. It's not that easy at all!