The Level Set Method applied to Structural Topology Optimization

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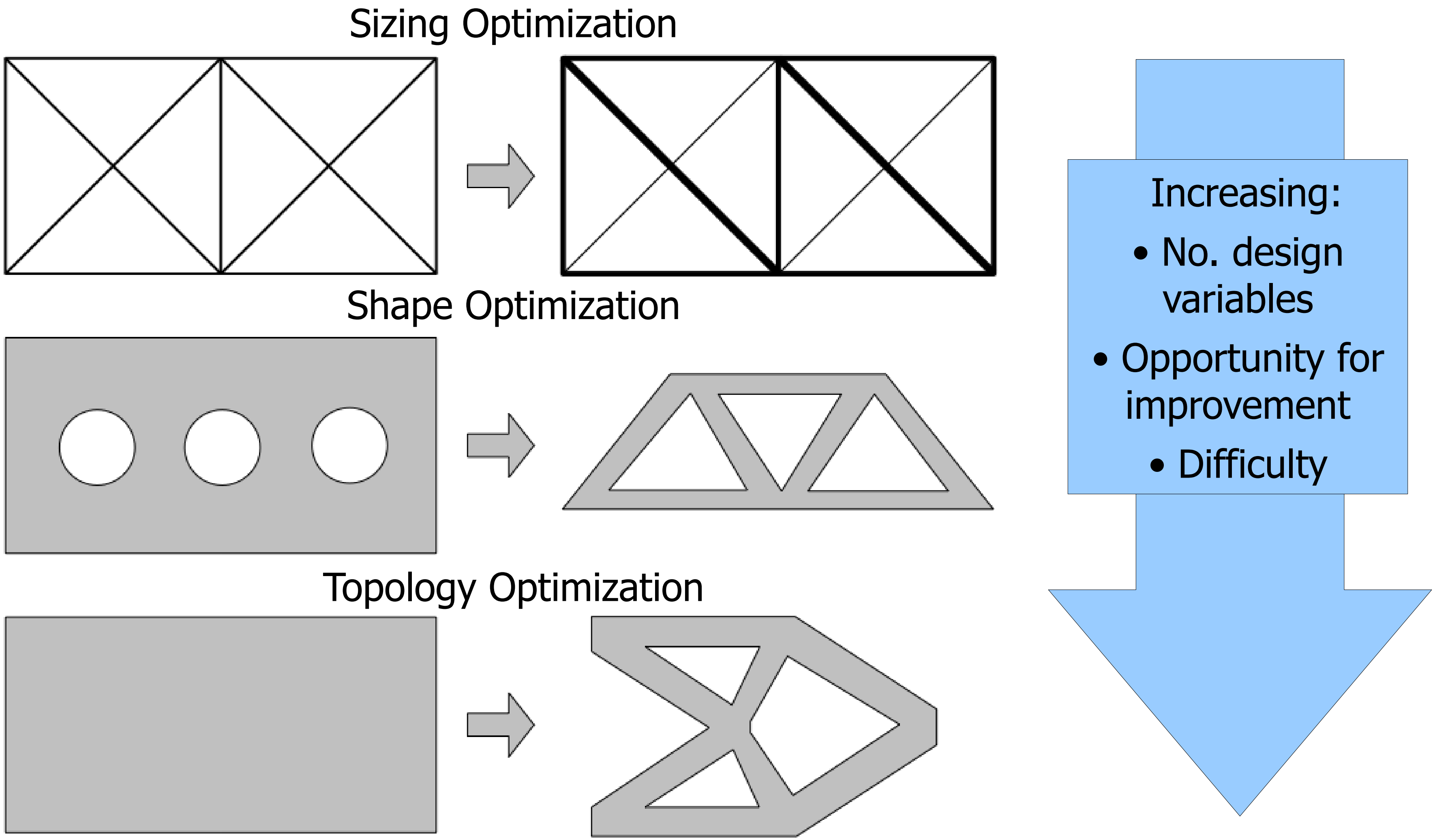
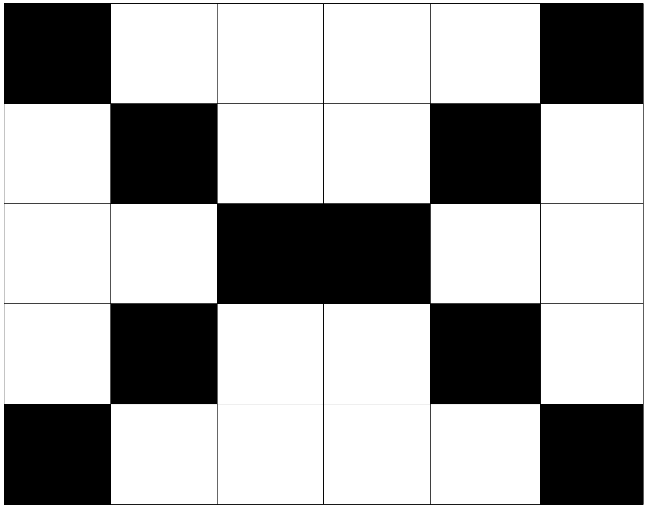
Structural Optimization

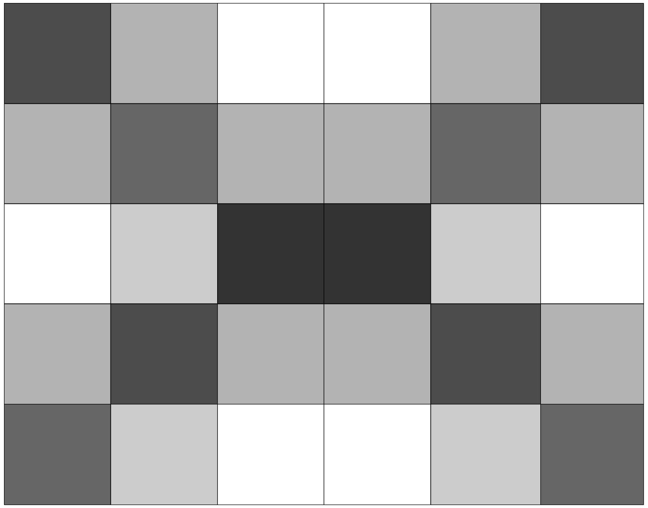


Brief Introduction to Topology Optimisation

- Optimal distribution of material
- Discretize design space using Finite Elements
- Relax 0-1 problem → Allow partially filled elements (variable density)



Discrete variables

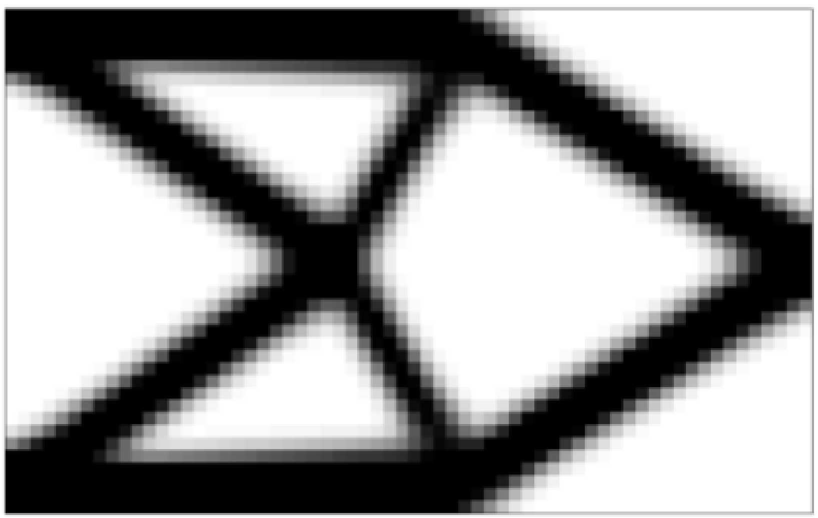


Continuous variables

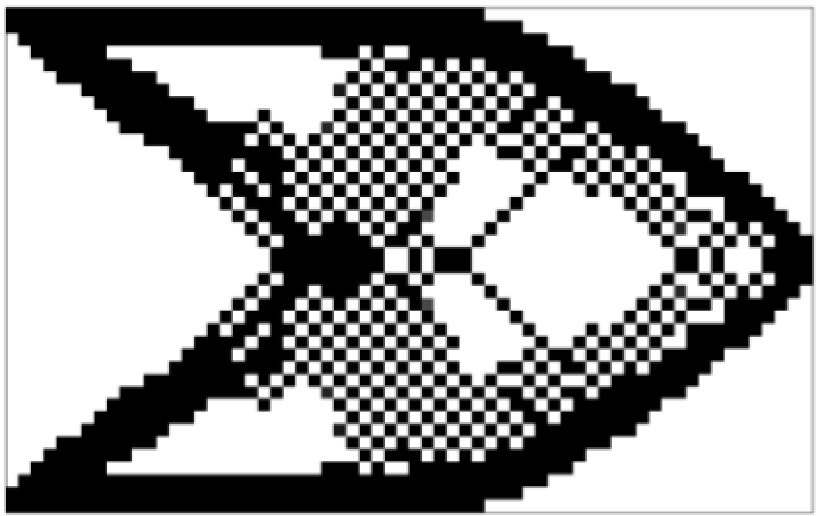
The SIMP Method

- Force variable density solutions towards discrete variable solution
- Penalise intermediate densities by raising to a power:

$$\rho^p, \rho \in \{0,1\}, p > 1$$



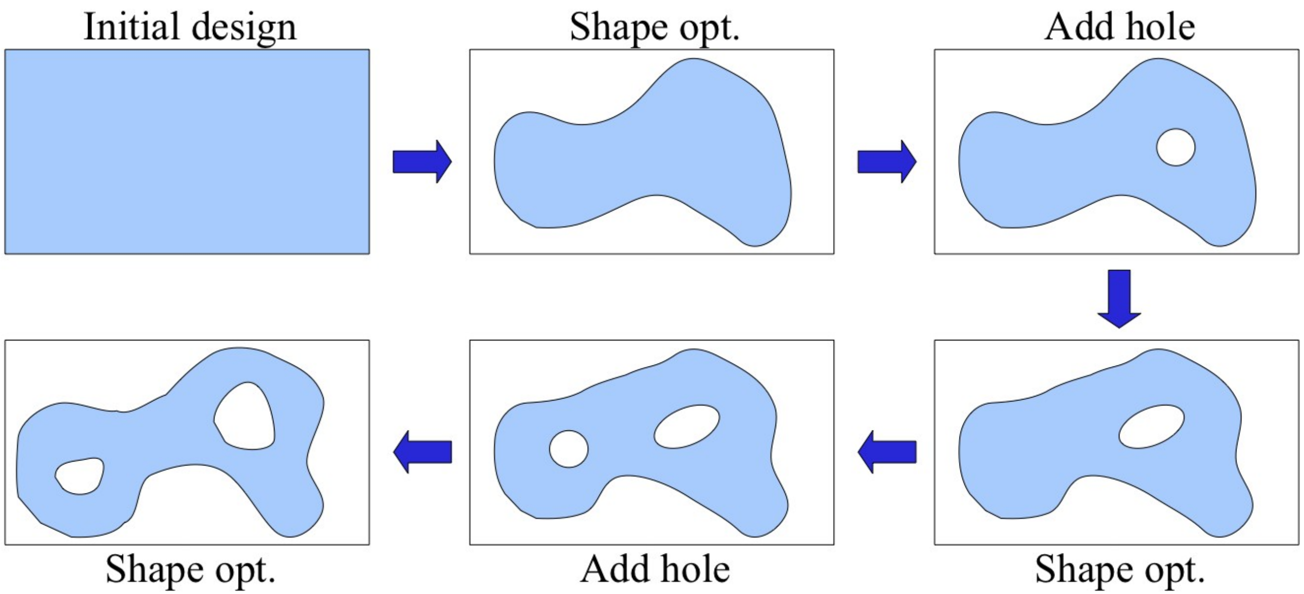
With sensitivity smoothing



No sensitivity smoothing

Boundary based Topology Optimization

- Problem: Element based methods.
  - results possess “fuzzy” boundaries & “grey” areas
- Solution: Boundary based methods.
  - combine shape optimization with provision to create/remove holes



Bubble Method: Eschenauer, Kobelev & Schumacher, 1994.

Spline Based Methods

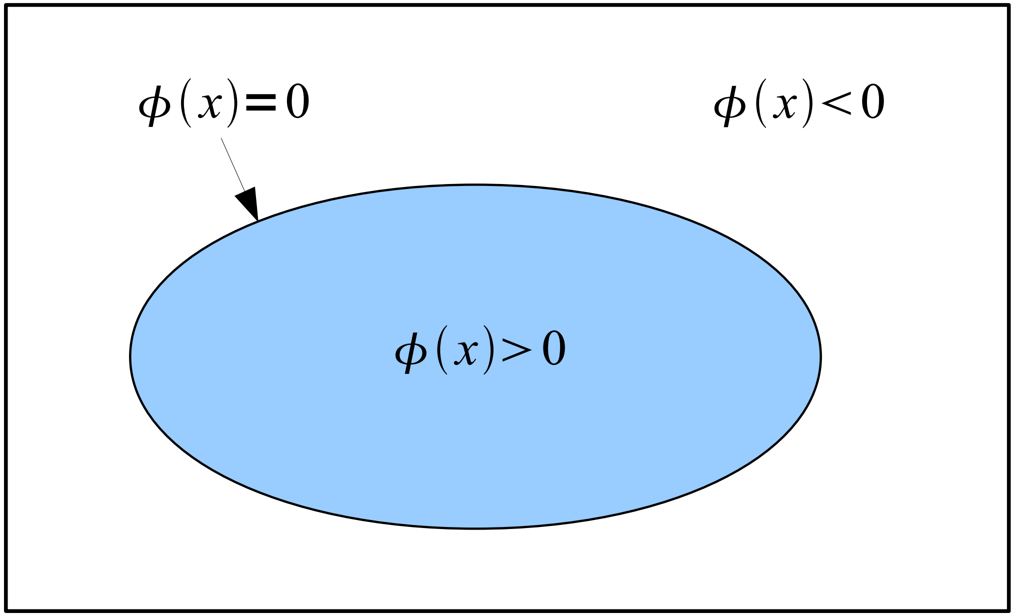
- Boundary represented using splines
- Design variables are control point positions
- Topological derivatives used to locate points to create new holes

However …

- Requires careful handling of splitting/merging boundaries
- Control points can bunch or spread → poor boundary representation
- Splines do not seem flexible enough for topology optimization

Implicit boundary representation

- Use a simple implicit function to define the boundary
- Assign a scalar value to each node of a discretized domain
- Boundary can naturally break and merge



$$\begin{cases} \phi(x) \geq 0, & x \in \Omega_S \\ \phi(x) = 0, & x \in \Gamma_S \\ \phi(x) < 0, & x \notin \Omega_S \end{cases}$$

Level set method

- Developed to compute motions of implicitly represented interfaces
- Numerical solution to:

$$\frac{\partial \phi(x,t)}{\partial t} + \nabla \phi(x,t) \frac{dx}{dt} = 0$$

- Discretize and rearrange:

$$\phi_i^{k+1} = \phi_i^k - \Delta t \left| \nabla \phi_i^k \right| V_{n,i}$$

Δt = time step

$V_{n,i}$ = velocity normal to boundary

i = grid point

k = current iteration

Level set based optimization

$$\phi_i^{k+1} = \phi_i^k - \Delta t |\nabla \phi_i^k| V_{n,i}$$

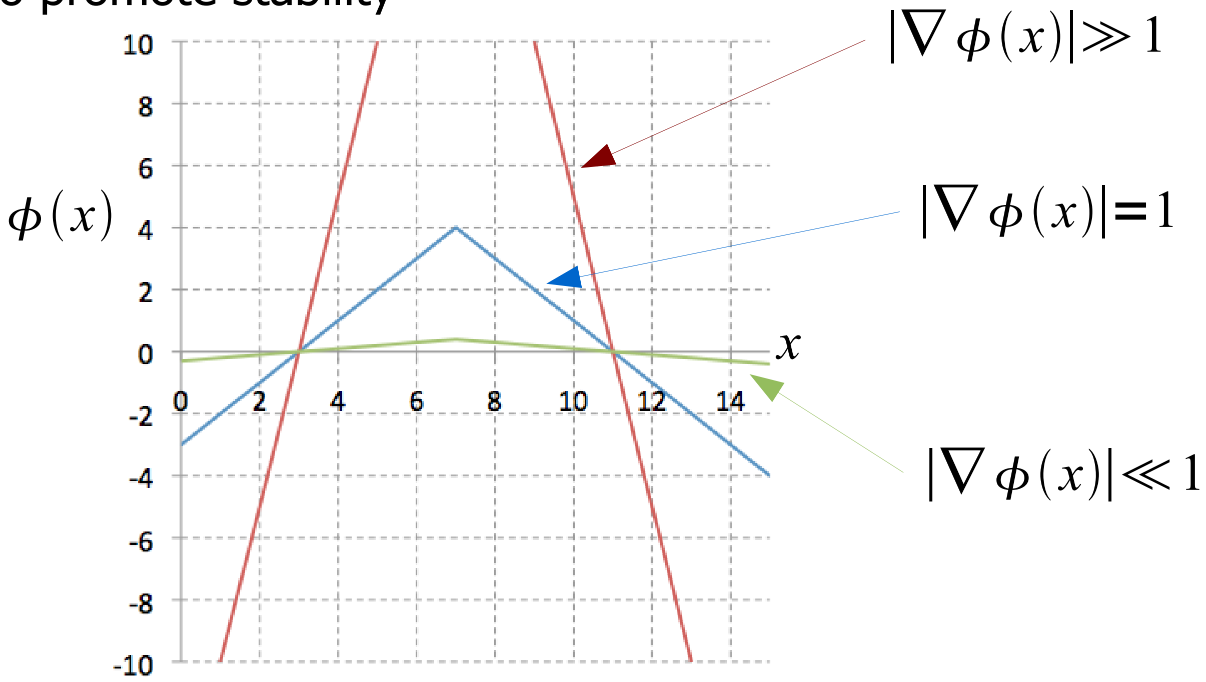
- Level set eq. can be used as an update rule in an optimization method
- Need to link the velocity function to the gradient of the objective
  - Shape sensitivity analysis
  - Decent method analogy

<u>Basic Algorithm</u>

1. Compute shape sensitivities along boundary
2. Define velocity function using descent method analogy
3. Move the boundary by numerically solving the level set equation
4. Repeat until a converged solution is found

Numerical considerations

- Analyse current structure to derive sensitivities → FEM
  - Fixed Grid (Eulerian) approach for efficiency
- Initialize $\phi(x)$ as signed distance function
  - Maintain to promote stability



Numerical considerations

- Stable time step defined by CFL condition: $\Delta t < |h/V|_{min}$
- Velocity function required at all grid nodes (not just at boundary)
    - Extension method to maintain signed distance function
    - Numerically solve: $\nabla \phi_t \cdot \nabla V_{ext} = 0$
- Spacial gradient computation:
    - Upwind scheme + WENO (stability & robustness)
- Narrow band approach for efficiency:
    - Only update $\phi(x)$ with certain distance of initial boundary
- Occasion re-initialization of $\phi(x)$ to a signed distance function

Compliance minimisation problem

- Objective to minimise structural compliance (or maximise stiffness)
- Upper limit on amount of available material
- Prescribed loading and boundary conditions

$$\min : C(u) = \int_{\Omega_s} A\,\varepsilon(u)\,\varepsilon(u)\,d\Omega_s \qquad \leftarrow \text{Total structural compliance}$$

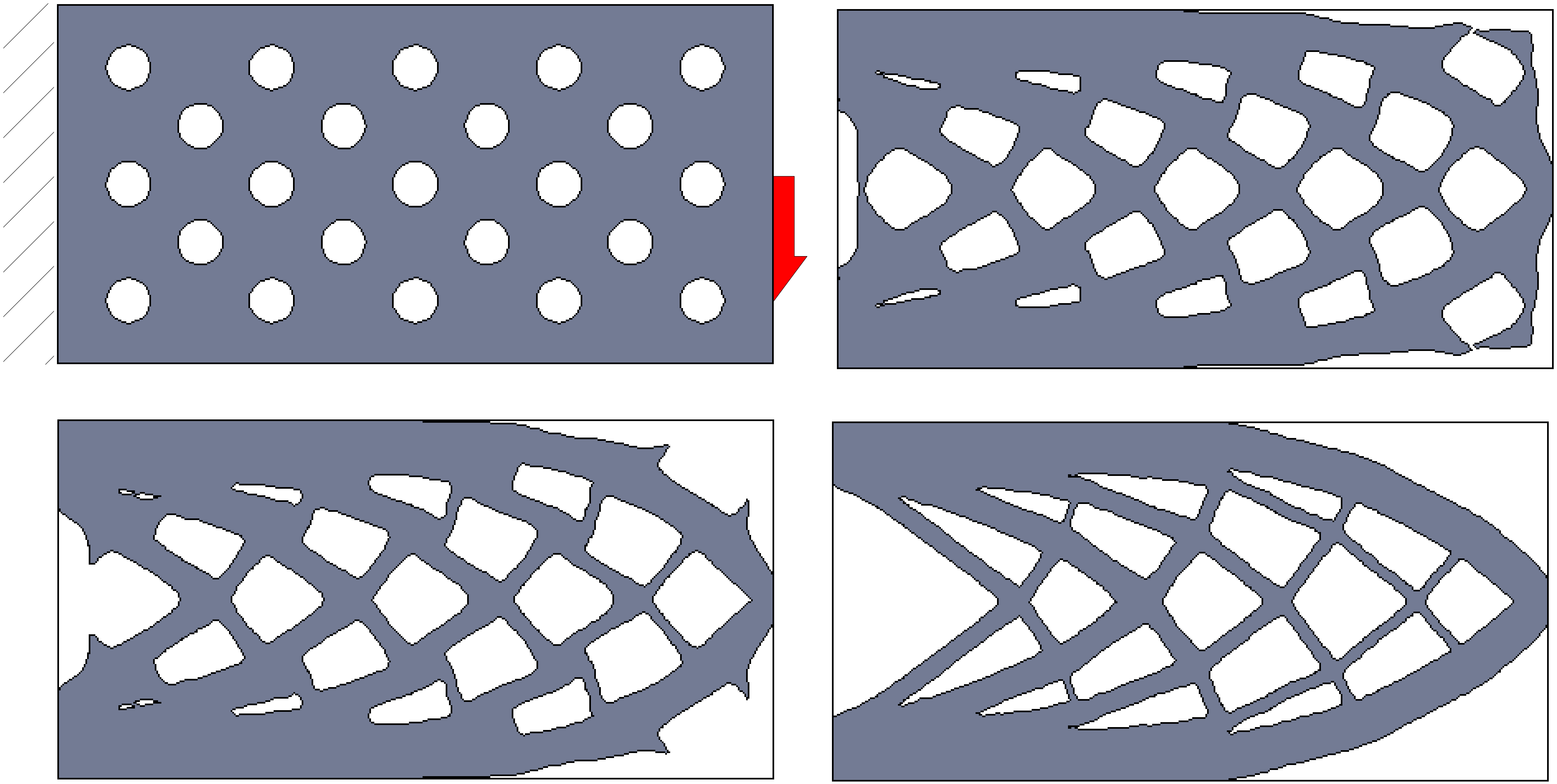
$$\text{s.t.} : \int_{\Omega_s} A\,\varepsilon(u)\,\varepsilon(v)\,d\Omega_s = \int_{\Gamma_s} fv\,ds \qquad \leftarrow \text{Static equilibrium}$$

$$\int_{\Omega_s} d\Omega_s \le Vol^* \qquad \leftarrow \text{Material volume constraint}$$

$$\min : \bar{C}(u) = C(u) + \lambda\left[\int_{\Omega_s} d\Omega_s - Vol^*\right] \qquad \leftarrow \text{Unconstrained problem}$$

$$\bar{C}'(u) = \int_{\Gamma_0} \left(A\,\varepsilon(u)\,\varepsilon(u) - \lambda\right) V_n\,d\Gamma_0 \qquad \leftarrow \text{Shape sensitivity}$$

Cantilever Beam Example

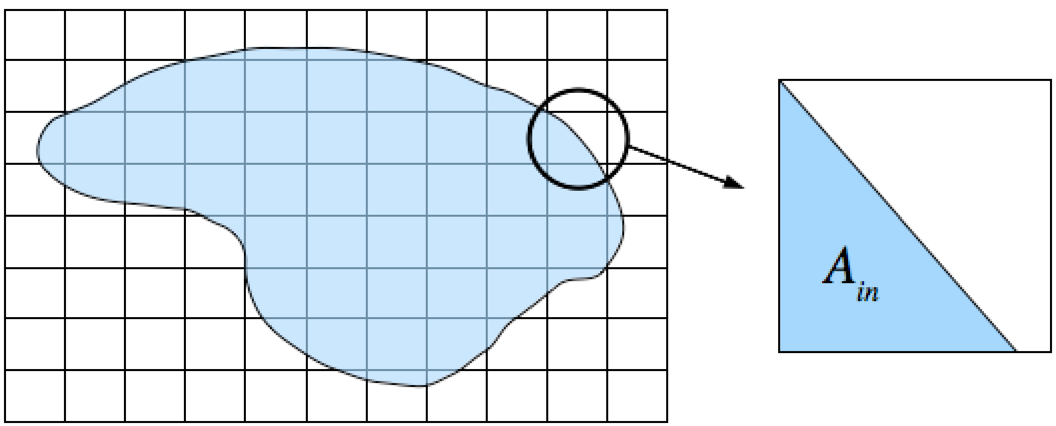


Challenges for Level Set Based Optimization

- Efficiency:
  - Constrained by CFL condition
  - Optimization only interested in final solution
- New hole creation:
  - Front tracking paradigm cannot create new holes
  - Solution is dependent on initial no. holes
- Accurate sensitivity computation:
  - Fixed grid elements do not match the boundary
  - Largest errors occur at the boundary
- Constraint handling:
  - Limited research to date – mainly concerned with a volume constraint

Fixed grid FEA

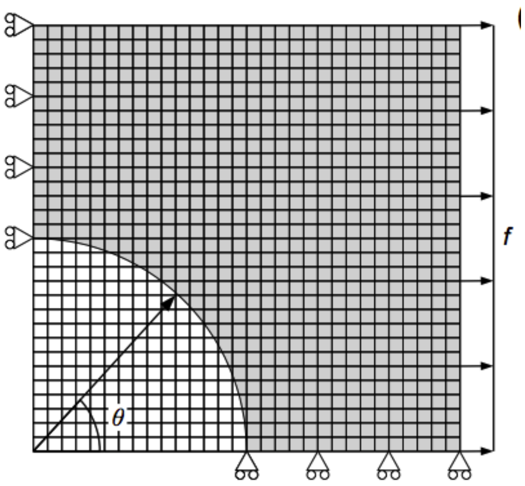
- "Topology" of FE mesh remains constant
- Structural boundary can cut though an element
- Discontinuous elements require special treatment
- Simplest approach to weight stiffness by a volume fraction

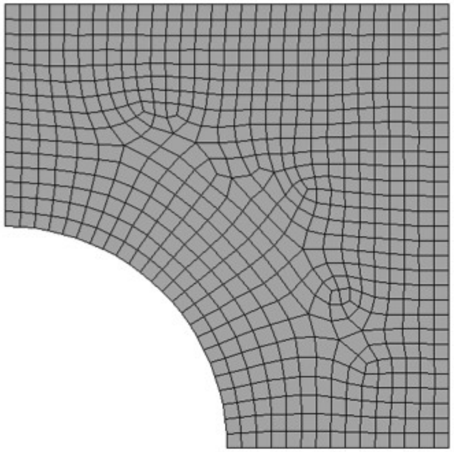


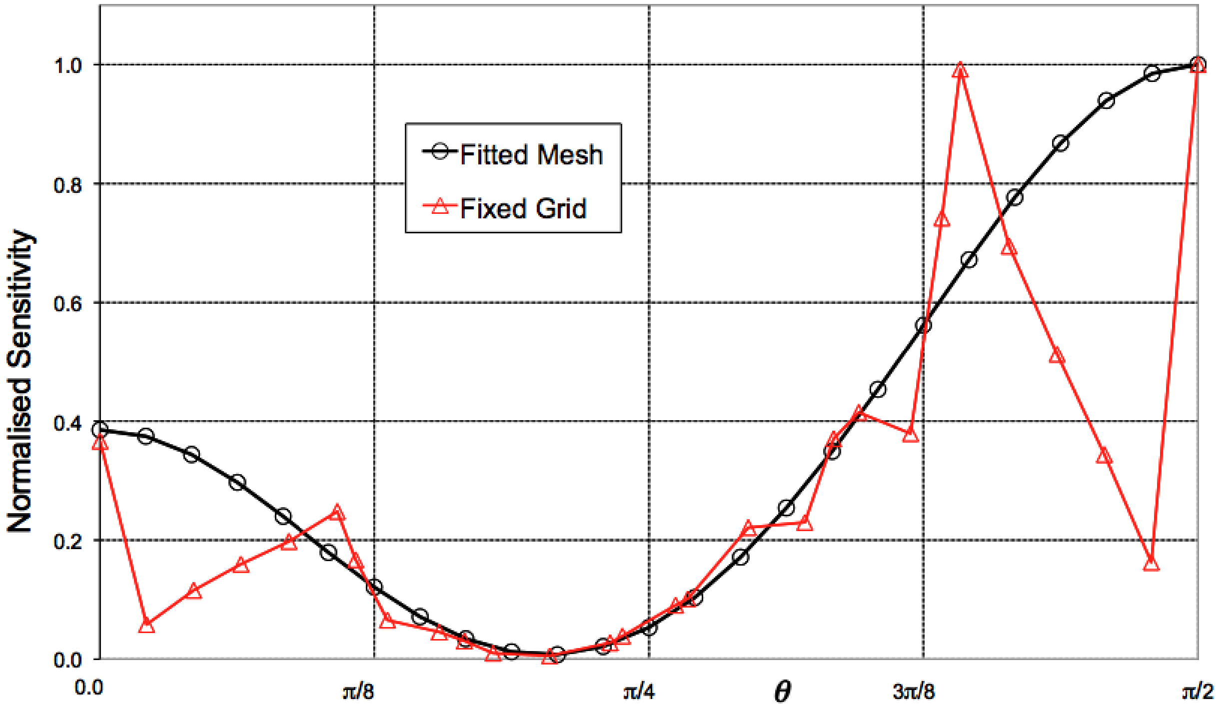
$$K_{cut} = \left(A_{in} / A_{full} \right) \times K_{full}$$

- Very efficient → Attractive property in structural optimization
- However, does not capture exact boundary position → Instability

Fixed grid sensitivity computation - example







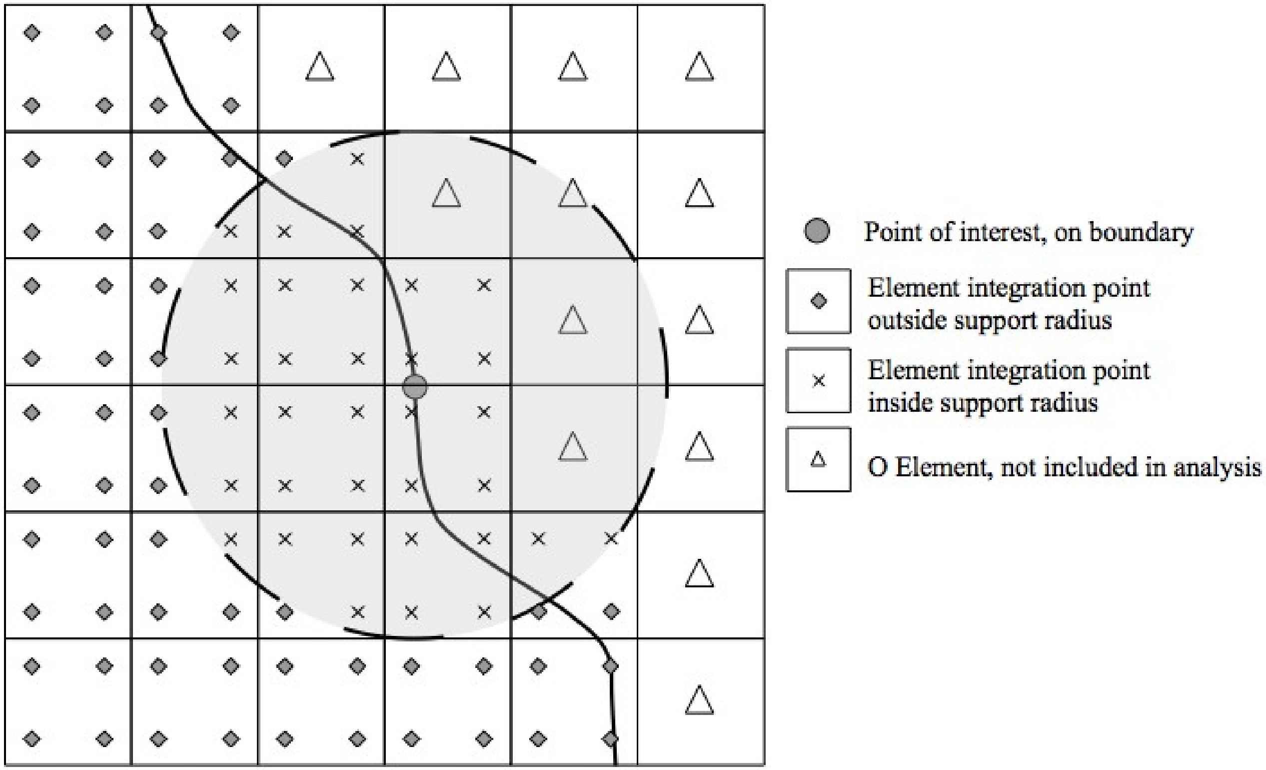
Sensitivities obtained using simple nodal averaging

Solution – Weighted Least Squares approach

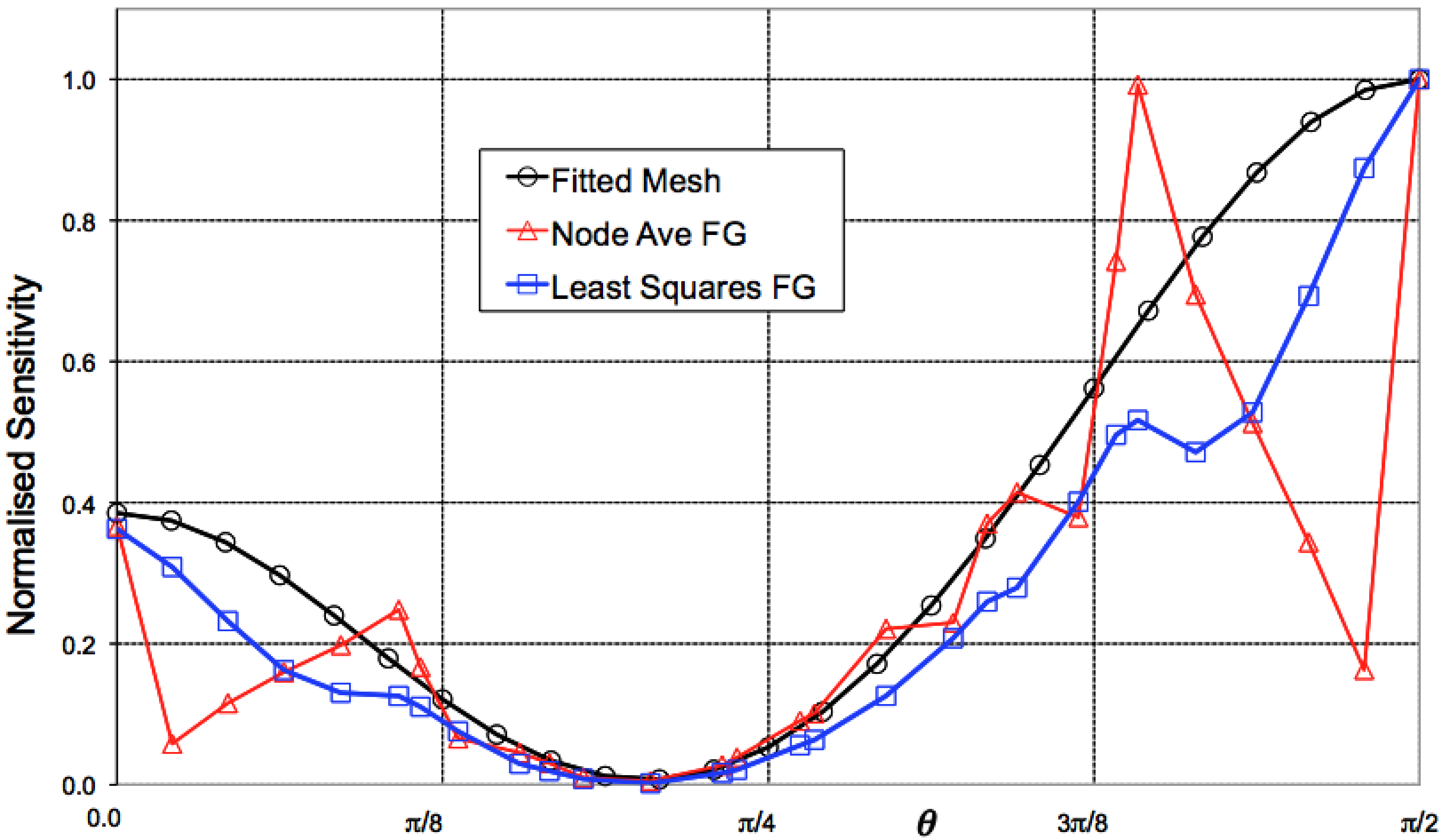
- Observations:
  - Generally, error in sensitivities increased as volume fraction decreased
  - Strains often more accurate when computed at gauss points
- Solution:
  - Compute sensitivities using a weighted least squares approach
- Numerical investigations:
  - Basis polynomial order
  - Support radius
  - Number of evaluations / element
  - Weighting function

Proposed Weighted LS sensitivity computation

- 2nd order basis function: $\varsigma(x, y) = c_0 + c_1 x + c_2 y + c_3 xy + c_4 x^2 + c_5 y^2$
- Weighted by volume fraction and inverse distance

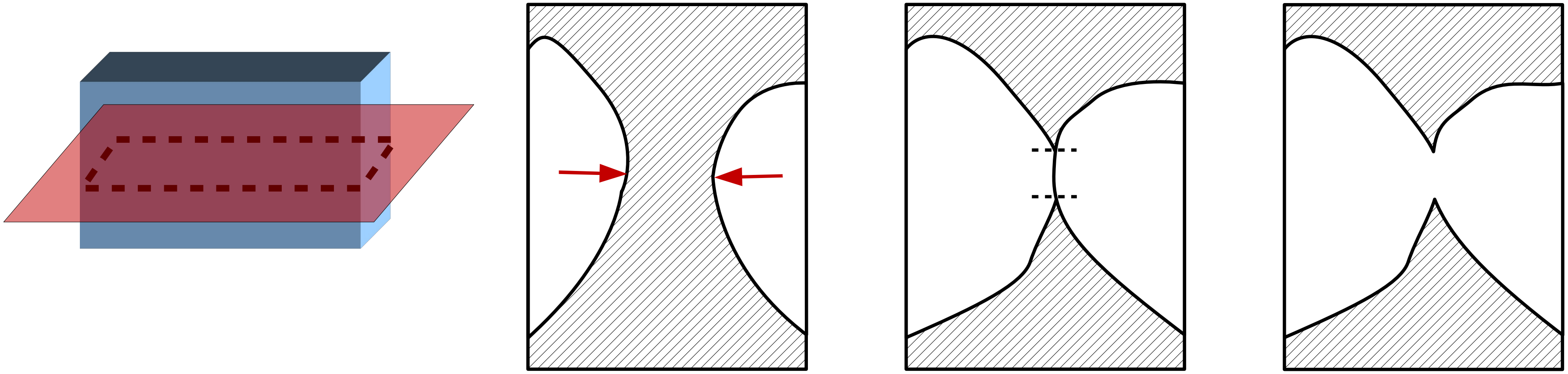


Fixed grid sensitivity computation example - revisited



Hole creation method

- Moving just the boundary does not allow new holes to emerge
- Could use topological derivatives:
  - Insert new hole every n iterations
  - Arbitrary, hole creation not linked to boundary movement
- New holes can emerge in 3D by an overlap of two approaching fronts



Top view: x-section

Hole creation method

- Exploit 3D hole creation mechanism in 2D:
  - Introduce 2nd implicit function to represent a pseudo 3rd dimension
  - Update 2nd function using shape sensitivities to define velocity

$$\phi_h^0(x) = \begin{cases} +\hat{h} & \text{if } x \in \Omega_s \\ -\hat{h} & \text{if } x \notin \Omega_s \end{cases}$$

← Initialize 2nd implicit function, $\hat{h}$ = *pseudo thickness*

$$\phi_h^{k+1}(x_i) = \phi_h^k(x_i) - \Delta t\, V_{n,i}$$

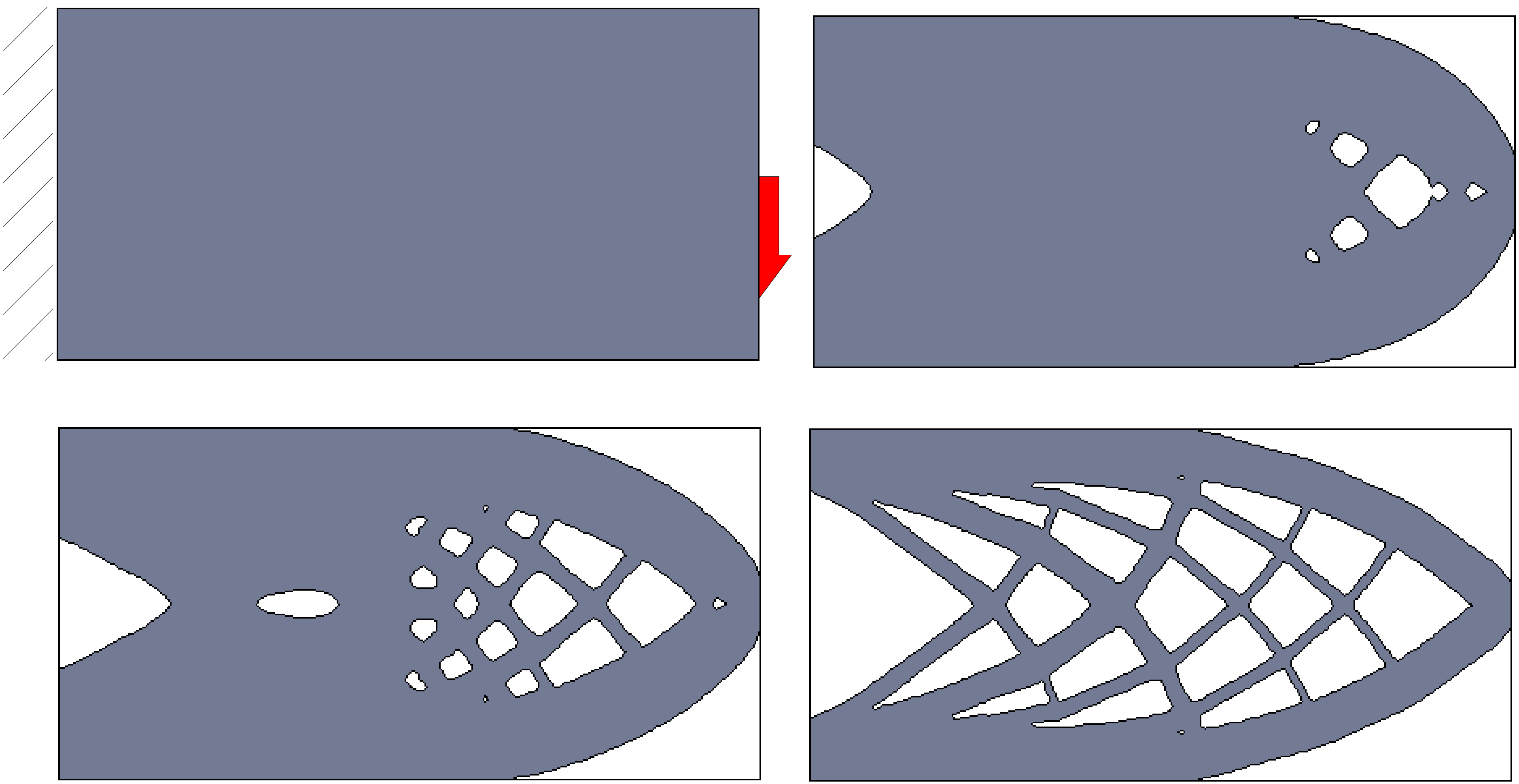
← Update 2nd implicit function

$$\phi_h^{k+1}(x_i) < 0 \quad \text{if } x \in \Omega_s$$

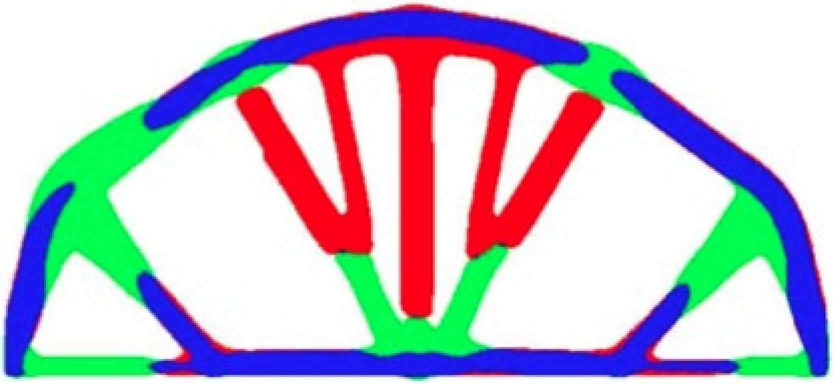
← Criteria for creating a new hole

- Holes can emerge naturally during optimization
- Hole creation is linked to boundary optimization

Hole creation method – Cantilever example

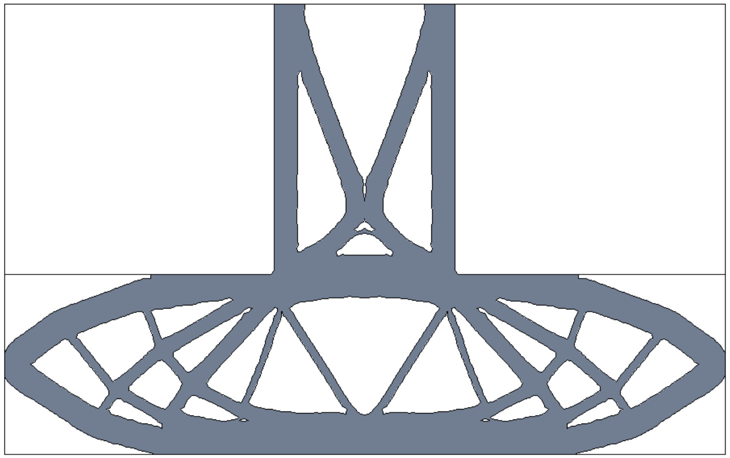


Applications



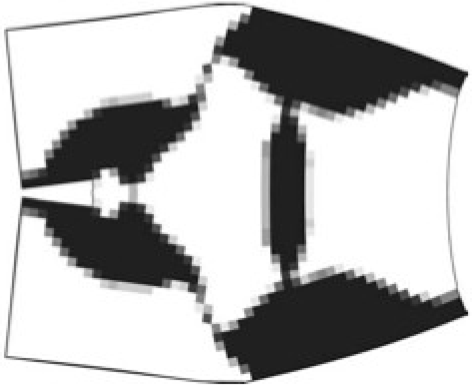
Multi-material

Wang MY & Wang X, Comput. Methods Appl. Mech. Engrg, 2004.



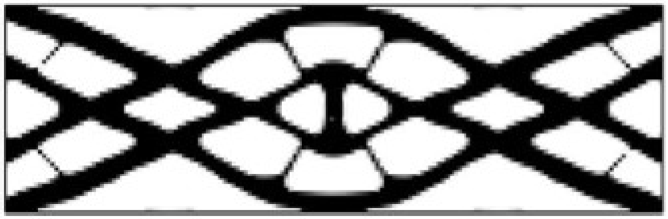
Robust optimization

Dunning PD et al, AIAA Journal, 2011.



Compliant mechanisms

Allaire G et al, J. Comp. Phys, 2004.



Eigenfrequency maximisation

Allaire G & Jouve F, Control and Cybernetics, 2005.

Future Work

- Multidisciplinary optimization (Aero-structural)
- Topology optimization of wing box structure (minimise weight)
- Heterogenous materials (composites, FGM)
- Realistic constraints:
  - Stress
  - Aeroelastic (divergence, flutter)

Any questions?

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